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Semi-Intuitionistic Logic with Strong Negation

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There is a well known interplay between the study of algebraic varieties and propositional calculus of various logics. Prime examples of this are boolean algebras and classical logic, and Heyting algebras and intuitionistic logic. After the class of Heyting algebras was generalized to the semi-Heyting algebras by H. Sankappanavar in [San08], its logic counterpart was developed by one of us in [Cor11] and further studied in [CV15].

Nelson algebras, or N-lattices were defined by H. Rasiowa [Ras58] to provide an algebraic semantics to the constructive logic with strong negation proposed by Nelson in [Nel49]. D. Vakarelov in [Vak77] presented a construction of Nelson algebras staring from Heyting ones. Applying this construction to semi-Heyting algebras, we obtained in [CV16] the variety of semi-Nelson algebras as a natural generalization of Nelson algebras. In this variety, the lattice of congruences of an algebra is determined through some of its deductive systems. Furthermore, the class of semi-Nelson algebras is arithmetical, has equationally definable principal congruences and has the congruence extension property.

It is the purpose of this work to present a Hilbert-style propositional calculus which is complete with respect to the algebras in this variety. Naming this logic *semi-intuitionistic logic with strong negation* was a natural choice. We believe that this logic will be of interest from the point of view of Many-Valued Logic, since its algebraic semantics show that it can provide many different interpretations for the implication connective. For example, on a chain with five elements, ten different semi-Nelson algebras may be defined, by changing the implication operation.

We present the algebraic motivation, defining semi-Nelson and semi-Heyting algebras. We then introduce the axioms and inference rule for the semi-intuitionistic logic with strong negation, together with some of their consequences. Finally we deal with completeness of the logic with respect to the class of semi-Nelson algebras, and present an axiomatic extension that has the variety of Nelson algebras as its algebraic semantics.

REFERENCIAS

[Cor11] Juan Manuel Cornejo. Semi-intuitionistic logic. *Studia Logica*, 98(1-2):9–25, 2011.

[CV15] Juan M. Cornejo and Ignacio D. Viglizzo. On some semi-intuitionistic logics. *Studia Logica*, 103(2):303–344, 2015.

[CV16] Juan M. Cornejo and Ignacio D. Viglizzo. Semi-nelson algebras. *Order*, 2016. Online. DOI: 10.1007/s11083-016-9416-x.

[Fit69] Melvin Chris Fitting. *Intuitionistic logic, model theory and forcing*. Studies in Logic and the Foundations of Mathematics. North-Holland Publishing Co., Amsterdam, 1969.

[Nel49] David Nelson. Constructible falsity. *J. Symbolic Logic*, 14:16–26, 1949.

[Ras58] H. Rasiowa. $\mathscr{N}$-lattices and constructive logic with strong negation. *Fund. Math.*, 46:61–80, 1958.

[San08] Hanamantagouda P. Sankappanavar. Semi-Heyting algebras: an abstraction from Heyting algebras. In *Proceedings of the 9th "Dr. Antonio A. R. Monteiro" Congress (Spanish)*, Actas Congr. "Dr. Antonio A. R. Monteiro", pages 33–66, Bahía Blanca, 2008. Univ. Nac. del Sur.

[Vak77] D. Vakarelov. Notes on $\mathcal{N}$-lattices and constructive logic with strong negation. *Studia Logica*, 36(1–2):109–125, 1977.

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A categorical equivalence for Nelson algebras

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The construction given by Kalman of Kleene algebras starting from distributive lattices, and by Vakarelov of Nelson algebras up from Heyting ones is generalized to obtain De Morgan algebras from distributive lattices. Necessary and sufficient conditions for these De Morgan algebras to be Nelson algebras are shown, and a characterization of the join-irreducible elements in the finite case is given. This construction is then used to establish an equivalence between the category of Nelson algebras and a category consisting of pairs of Heyting algebras and one of their filters.

Synonymy and categories of logics

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The study of categories of logics is motivated, among other reasons, by questions such as how to combine logics and when they are equivalent (see for example [Arn15] and [Cal07] and the bibliography therein). Categories of logics do not always have good categorical properties such as the existence of finite limits and colimits. One natural way to obtain from these categories new ones with better properties is to consider the quotient category induced by the interdemonstrability relation, as done in [MaM16].

Following the aforementioned works, we consider the category whose objects are Tarskian logics and whose morphisms are flexible translations that preserve the synonymy relation between formulas, in the sense of [Smi62]. This relation induces a congruence on the class of morphisms of this category.

In this communication, we study the quotient category induced by synonymy and some of its categorical properties.

REFERENCIAS

[Arn15] P. Arndt. *Homotopical categories of logics*. The Road to Universal Logic, Festschrift for the 50th Birthday of Jean-Yves Béziau Volume I, (2015), 13-58.
[Cal07] C. Caleiro, R Gonçalves. *Equipollent logical systems*. Logica Universalis: Towards a General Theory of Logic (editor: J.-Y. Béziau), (2007), 97-110.
[MaM16] C. Mendes, H. Mariano. *Towards a good notion of categories of logics*. Preprint arXiv:1404.3780v2.
[Smi62] T. Smiley. *The independence of connectives*. Journal of Symbolic Logic 27, (1962), 426-436.

On the logic that preserves degrees of truth associated to involutive Stone algebras

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Let A be a De Morgan algebra. Denote by $K(A)$ the set of all elements $a \in A$ such that the De Morgan negation of a, $\neg a$, coincides with the complement of a. For every $a \in A$, let $K_a = \{k \in K(A) : a \leq k\}$ and, if K_a has a least element, denote by ∇a the least element of K_a. The class of all De Morgan algebras A such that for every $a \in A$, ∇a exists, and the map $a \mapsto \nabla a$ is a lattice–homomorphism is called the class of involutive Stone algebras, denoted by **S**. These algebras were introduced by Cignoli and Sagastume ([1]) in connection to the theory of n-valued Łukasiewicz–Moisil algebras.

In this work, we focus on the logic that preserves degrees of truth $\mathbb{L}_{\mathbf{S}}^{\leq}$ associated to involutive Stone algebras. This follows a very general pattern that can be considered for any class of truth structure endowed with an ordering relation; and which intend to exploit manyvaluedness focussing on the notion of inference that results from preserving lower bounds of truth values, and hence not only preserving the value 1 (see [3, 4, 5]).

Among other things, we prove that $\mathbb{L}_{\mathbf{S}}^{\leq}$ is a manyvalued logic (with six truth values) that can be determined by a finite number of matrices (four matrices). Besides, we show that $\mathbb{L}_{\mathbf{S}}^{\leq}$ is a paraconsistent logic, moreover, we prove that it is a genuine LFI (Logic of Formal Inconsistence, [2]) with a consistence operator that can be defined in terms of the original set of connectives. Finally, we study the proof theory of $\mathbb{L}_{\mathbf{S}}^{\leq}$ providing a Gentzen calculus for it, which is sound and complete with respect to the logic.

Referencias

[1] R. Cignoli and M. S. de Gallego, *Dualities for some De Morgan álgebras with operators and Łukasiewicz algebras*, J. Austral Math. Soc (Series A), 34 (1983), 377-393.

[2] Carnielli, W.A., Coniglio, M.E. and Marcos, J., Logics of Formal Inconsistency. In: *Handbook of Philosophical Logic*, vol. 14, pp. 15–107. Eds.: D. Gabbay; F. Guenthner. Springer, 2007.

[3] J. M. Font. On substructural logics preserving degrees of truth. Bulletin of the Section of Logic, 36, 117-130, 2007.

[4] J. M. Font. Taking degrees of truth seriously. Studia Logica (Special issue on Truth Values, Part I), 91, 383-406, 2009.

[5] J. M. Font, A. Gil, A. Torrens, and V. Verdú. On the infinite-valued Lukasiewicz logic that preserves degrees of truth. Archive for Mathematical Logic, 45, 839-868, 2006.

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Una dualidad para semirretículos distributivos monótonos

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Las lógicas modales monótonas basadas en la lógica clásica o en la lógica intuicionista son generalizaciones de la lógica modal normal (clásica o intuicionista) en las cuales el axioma $m(\varphi \to \psi) \to (m\varphi \to m\psi)$ se debilita a una condición de monotonía, que puede ser expresada como un axioma $m(\varphi \wedge \psi) \to m\varphi$ o una regla (de $\varphi \to \psi$ se deriva $m\varphi \to m\psi$).

En el trabajo que presentaremos hemos estudiado una dualidad estilo Stone para los $\wedge$-semirretículos distributivos con último elemento (en adelante semirretículos) dotados de un operador modal monótono. Los semirretículos se encuentran presentes en diversas estructuras algebraicas relacionadas al estudio de las lógicas no clásicas, por ejemplo en la semántica algebraica del fragmento $\{\rightarrow, \wedge, \top\}$ de la lógica intuicionista que es la variedad de los semirretículos implicativos.

Las extensiones canónicas fueron introducidas por Jonsson y Tarski para estudiar las algebras de Boole con operadores. El propósito principal era identificar qué forma debía tener el dual de una operación adicional. La teoría de las extensiones canónicas ha sido simplificada y generalizada para ser aplicada en estructuras algebraicas mas allá de las álgebras de Boole. En nuestro trabajo usamos la extensión canónica como una herramienta algebraica para representar los operadores modales por medio de relaciones en el espacio dual. La mayoría de los resultados de este trabajo son aplicables, bajo menores modificaciones, al estudio de los retículos distributivos acotados, semirretículos implicativos, álgebras de Heyting y álgebras de Boole con operadores monótonos.

REFERENCIAS

[1] Celani S.A.: Topological representation of distributive semilattices. Sci. Math. Japonicae Online. 8, 41–51 (2003)

[2] Dunn, J.M., Gehrke, M., Palmigiano, A.: Canonical extensions and relational completeness of some substructural logics. J. Symbolic Logic. 70 (3), 713–740 (2005)

[3] Hansen, H.H.: Monotonic modal logics. Master's thesis, University of Amsterdam, Faculty of Science, Institute for Logic, Language and Computation, (2003)

Congruencias que preservan aniquiladores en DN-álgebras

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Una *DN-álgebra* es un supremo-semirretículo con último elemento el cual cada filtro principal es un retículo distributivo acotado. Las DN-álgebras forman una variedad y generalizan tanto a las álgebras de Tarski como a los retículos distributivos acotados. Varios autores han estudiado estas estructuras en [5], [6] y [4]. Recientemente en [2], se desarrolla una dualidad topológica estilo Stone para la clase de las DN-álgebras. Como aplicación, dicha dualidad nos permite traducir y resolver en términos topológicos problemas algebraicos. Un buen ejemplo es la caracterización de las congruencias de una DN-álgebra a través de *N-subespacios* del espacio asociado a la DN-álgebra.

Por otro lado, en [7], se introduce la noción de *congruencia que preserva aniquiladores*, o *AP-congruencia*, en un retículo distributivo acotado A como una congruencia θ tal que para todo $a, b \in A$, si $a \wedge b \equiv_\theta 0$ entonces existe $c \in A$ tal que $a \wedge c = 0$ y $c \equiv_\theta b$. En [3] se obtienen nuevas equivalencias y una interpretación topológica de esta noción. Siguiendo los resultados desarrollados en [2] y [1], el objetivo de esta comunicación es extender el concepto de AP-congruencia a la clase de las DN-álgebras. Luego, representamos a las AP-congruencias a través de N-subespacios cumpliendo una condición adicional.

REFERENCIAS

[1] Calomino I.; Celani S.: *A note on annihilators in distributive nearlattices*. Miskolc Mathematical Note **16** (2015), 65-78.
[2] Celani S.; Calomino I.: *Stone style duality for distributive nearlattices*. Algebra Universalis **71** (2014), 127-153.
[3] Celani S.: *Remarks on annihilators preserving congruence relations*. Mathematica Slovaca **62** (2012), 389-398.
[4] Chajda I.; Halaš R.; Kühr J.: *Semilattice Structures*. Research and Exposition in Mathematics, Heldermann Verlag (2007).
[5] Cornish W.; Hickman R.: *Weakly distributive semilattices*. Acta Math. Acad. Sci. Hungar. **32** (1978), 5-16.
[6] Hickman R.: *Join algebras*. Communications in Algebra **8** (1980), 1653-1685.
[7] Janowitz M.: *Annihilator preserving congruence relations of lattices*. Algebra Universalis **5** (1975), 391-394.

Selfextensional logics with a nearlattice term

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The aim of this communication is to propose a definition of when a ternary term m of an algebraic language $\mathscr{L}$ is a distributive nearlattice term (DN-term for short) for a sentential logic §. Then, we show that selfextensional logics with a distributive nearlattice term m can be characterized as the sentential logics § for which there exists a class of algebras K such that the $\{m\}$-reducts of the algebras of K are distributive nearlattices and the consequence relation of § can be defined using the order induced by the term m on the algebras of K.

The notion of distributive nearlattice can be defined in two equivalent way. An algebra $\langle A,m\rangle$ of type (3) is a distributive nearlattice if satisfies some identities; and a ternary algebra $\langle A,m\rangle$ is a distributive nearlattice if and only if A with the binary term $x \vee y := m(x,x,y)$ is a join-semilattice such that for every $a \in A$, the upset $[a)$ is a distributive lattice with respect to the order induced by $\vee$.

REFERENCIAS

[1] I. Chajda, R. Halaš, and J. Kühr. *Semilattice structures*, volume 30. Heldermann, Lemgo, 2007.
[2] J. M. Font and R. Jansana. *A general algebraic semantics for sentential logics*, volume 7 of *Lecture Notes in Logic*. The Association for Symbolic Logic, 2 edition, 2009.
[3] L. González. The logic of distributive nearlattices. *Submitted to Soft Computing*, 2016.
[4] R. Hickman. Join algebras. *Communications in Algebra*, 8(17):1653–1685, 1980.
[5] R. Jansana. Selfextensional logics with implication. In J-Y. Beziau, editor, *Logica Universalis*, pages 65–88. Springer, 2005.
[6] R. Jansana. Selfextensional logics with a conjunction. *Studia Logica*, 84(1):63–104, 2006.

L_n^2-álgebras

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Teniendo en cuenta que la variedad $\mathscr{L}_n^2$ está constituida por las álgebras de Łukasiewicz m–generalizadas de orden n considerando $m = 2$. Es decir, las L_n^2–álgebras en las cuales $f^4x = x$. Mostramos propiedades de los átomos que serán de gran utilidad para describir detalladamente a las álgebras simples de $\mathscr{L}_n^2$. Más precisamente, hallamos el número de L_n^2–álgebras simples y su cardinal.

REFERENCIAS

[1] T. Blyth, J. Varlet, Ockham Algebras. Oxford University Press, New York, 1994.

[2] R. Cignoli, *Moisil Algebras*, Notas de Lógica Matemática 27, Univ. Nacional del Sur, Bahía Blanca, Argentina, 1970.

[3] A. V. Figallo, C. Gallardo, A. Ziliani, Weak implication on generalized Łukasiewicz algebras of order n, Bull. Sect. Logic Univ. Lódź 39, 4 (2010), 187–198.

[4] J. Vaz De Carvalho and T. Almada, A generalization of the Lukasiewicz algebras, Studia Logica 69 (2001), 329-338.

[5] J. Vaz De Carvalho, On the variety of $m-$generalized Łukasiewicz algebras of order n, Studia Logica 94 (2010), 291–305.

A topological duality for tense LM$_n$-algebras and applications

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In 2007, tense n-valued Łukasiewicz–Moisil algebras (or tense LM_n-algebras) were introduced by Diaconescu and Georgescu as an algebraic counterpart of the tense n-valued Moisil logic. In this paper we continue the study of the tense LM_n-algebras initiated by Figallo and Pelaitay in [1]. More precisely, we determine a topological duality for these algebras. This duality enable us not only to describe the tense LM_n-congruences on a tense LM_n-algebra, but also to characterize the simple and subdirectly irreducible tense LM_n-algebras. Furthermore, by means of the aforementioned duality a representation theorem for tense LM_n-algebras is proved, which was formulated and proved by a different method by Georgescu and Diaconescu in [2].

REFERENCIAS

[1] A. V. Figallo and G. Pelaitay. Discrete duality for tense Łukasiewicz–Moisil algebras. Fund. Inform., **136** (4), 317–329, (2015).

[2] D. Diaconescu and G. Georgescu, Tense operators on MV-algebras and Łukasiewicz–Moisil algebras, *Fund. Inform.* **81** (4), 379–408, (2007).

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