Eingenvector descomposition of trees

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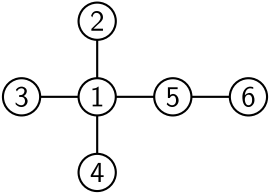
UMA. 22 de Septiembre, 2016

Contents

$Supp(x)$



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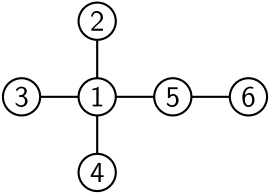

$$x = (0, 1, 0, -2, -5, 10)^T$$

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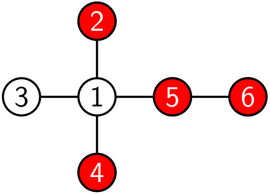


$$x = (0, 1, 0, -2, -5, 10)^T$$

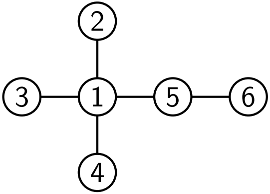
Known results

Lemma (Sander and Sander, 2009)

Given a graph G, and an eigenvalue λ of G, let $\mathcal{B} := \{b_i, \ldots, b_k\}$ a base of $\mathcal{E}_\lambda$, then

$$\mathcal{S}upp(\mathcal{E}_\lambda) = \mathcal{S}upp(\mathcal{B})$$

$Supp(T)$



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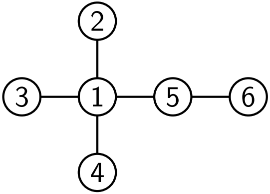

$$\mathcal{B}_{\mathcal{N}(T))} = \left\{ \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \\ 0 \\ 0 \end{pmatrix} \right\}$$

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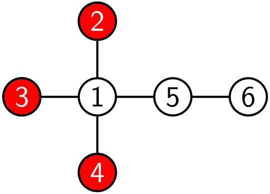

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New results

Lemma

Let T be a tree, and $v \in \mathcal{S}upp(T)$, then

$$N(v) \cap \mathcal{S}upp(T) = \emptyset$$

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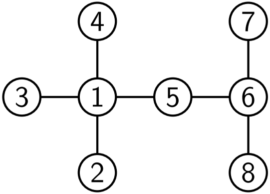
Corollary

Let T be a tree, then $\mathcal{S}upp(T)$ is an independent set of T.

Definitions



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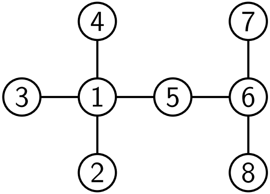

$$\mathcal{S}upp(T) = \{2, 3, 4, 5, 7, 8\}$$

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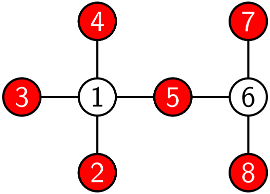

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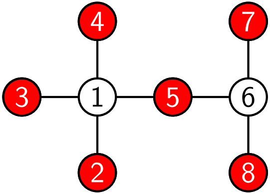

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$$N(\mathcal{S}upp(T)) = \{1, 6\} = \mathcal{C}ore(T)$$

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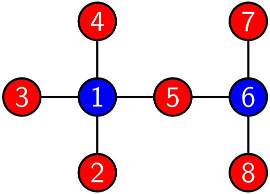

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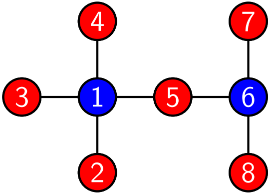

$$Supp(T) = \{2, 3, 4, 5, 7, 8\}$$

$$N(Supp(T)) = \{1, 6\} = Core(T)$$

$$N\left[Supp(T)\right] = V(T)$$

Aplications

Theorem

Let T be a S-tree, then $\alpha(T) = \left|\mathcal{S}upp(T)\right|$ and $\mathcal{S}upp(T)$ is the unique maximum independent set of T.

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Theorem

If T is a S-Tree, then $\nu(T) = \tau(T) = |\mathcal{C}ore(T)|$. Furthermore $\mathcal{C}ore(T)$ is the unique minimum vertex cover of T, and for all $M \in \mathcal{M}(T)$, and for all $e \in M$ we have that $|e \cap \mathcal{C}ore(T)| = 1$.

Introduction

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A tree T is an N-tree if T is non-singular

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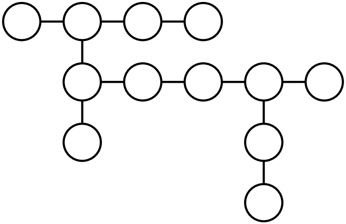

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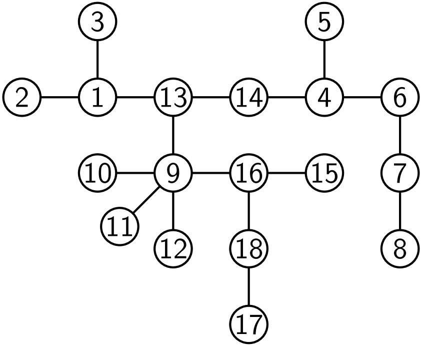

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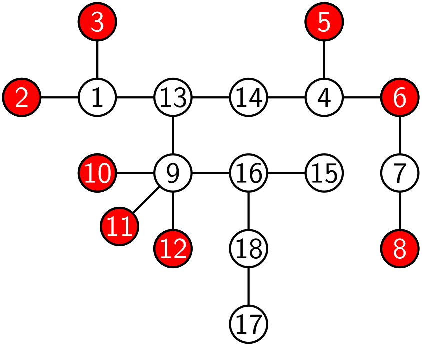

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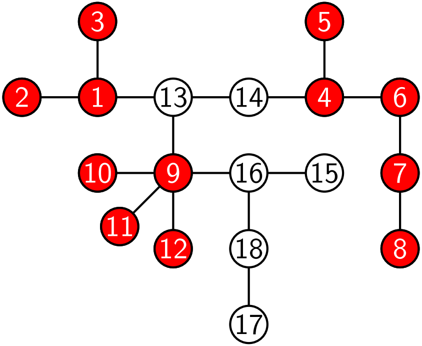
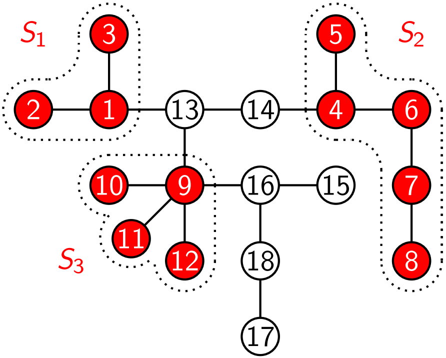
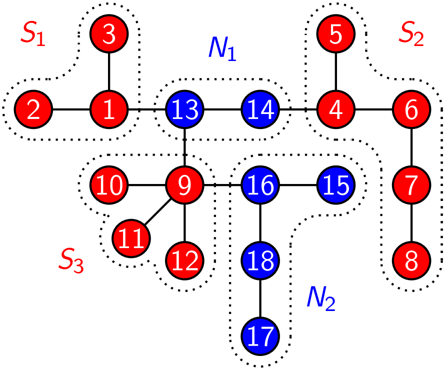


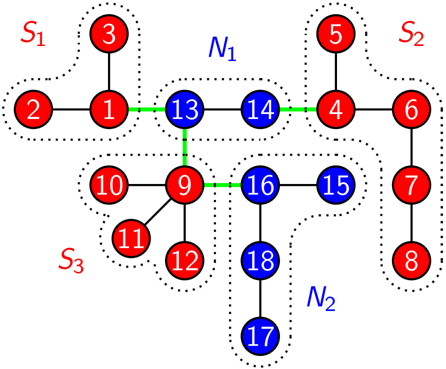
S forest



N forest



Conection edges



$\mathcal{F}_S(T)$

Theorem

For any tree T, the S-forest, $\mathcal{F}_S(T)$, is a forest of S-trees.

$\mathcal{N}(T)$

Lemma

For any tree T, and $H \in \mathcal{F}_S(T)$

1. $\mathcal{S}upp(H) = \mathcal{S}upp(T) \cap V(H)$
2. $\mathcal{C}ore(H) = N(\mathcal{S}upp(T)) \cap V(H)$
3. If $x \in \mathcal{N}(T)$, then $x\downarrow_H^T \in \mathcal{N}(H)$.
4. If $x \in \mathcal{N}(H)$, then $x\uparrow_H^T \in \mathcal{N}(T)$.

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For any tree T, *and* $H \in \mathcal{F}_S(T)$

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2. $\mathcal{C}ore(H) = N(\mathcal{S}upp(T)) \cap V(H)$
3. *If* $x \in \mathcal{N}(T)$, *then* $x\downharpoonright_H^T \in \mathcal{N}(H)$.
4. *If* $x \in \mathcal{N}(H)$, *then* $x\upharpoonright_H^T \in \mathcal{N}(T)$.

Corollary

For any tree T

$$\mathcal{N}(T) = \bigoplus_{S \in \mathcal{F}_H(T)} \mathcal{N}(H)$$

Conection edges

Lemma

For any tree T there exists a maximum matching M of T such that $M \cap CE(T) = \emptyset$.

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Lemma

Given a tree T, and any $e \in CE(T)$, for each $u \in \mathcal{C}ore(S_e)$ and for each $v \in V(N_e)$ we have that $rank(T) = rank(T_{uv})$.

N-forest

Theorem

For any tree T, the N-forest, $\mathcal{F}_N(T)$, is a forest of N-trees.

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Let T be a tree, then

$$\alpha(T) = \frac{|V(\mathcal{F}_N(T))|}{2} + |\mathcal{S}upp(T)|$$

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Corollary

For any tree T:

$$m(T) = \prod_{H \in \mathcal{F}_S(T)} m(H)$$

¡Gracias!