Characterizing Probe Unit Interval Graphs within the Class of Interval Graphs

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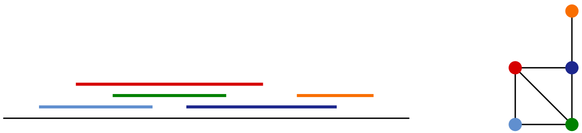

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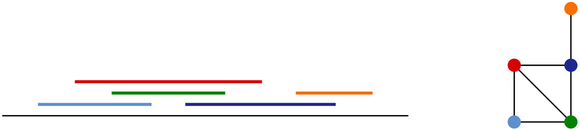


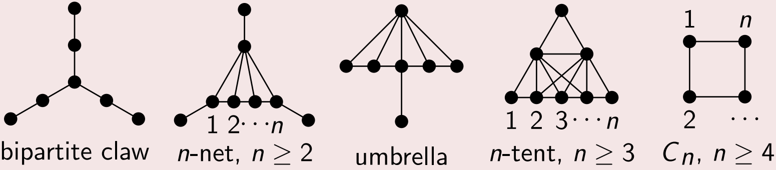
- The class of interval graphs is a hereditary class by induced subgraph.

Characterization of Interval Graphs

Theorem (Boland and Lekkerkerker, 1962)

Given a graphs G, the following conditions are equivalent:

1. The graph G is an interval graph.
2. The graph G is chordal and contains no asteroidal triple. (*)
3. The graph G does not contain any of the following graphs as induced subgraphs.



- (*) Three vertices in a graph G form an asteroidal triple if every two of them are connected by a path avoiding the third one.

Unit interval graphs and Proper interval graphs

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Theorem (Roberts, 1969)

Given an interval graph G the following conditions are equivalent:

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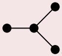
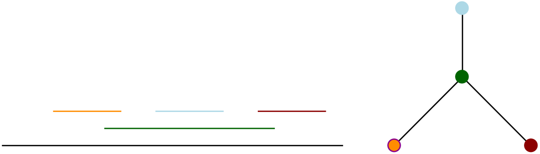
3. G contains no induced claw .



Probe Classes

- Let $G = (V, E)$ be a graph. A probe-$\mathcal{G}$ graph is a graph whose vertex set can be partitioned into two sets: a set P of probe vertices and an independent set N (a set of pairwise nonadjacent vertices) of nonprobe vertices in such a way that a graph $G^* = (N \cup P, E \cup F)$ in $\mathcal{G}$ can be obtained by adding a set (possibly empty) F of edges with both endpoints in N. Such a graph G^* is called a probe-$\mathcal{G}$ completion of G.

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- If a partitioned graph $G = (P \cup N, E)$ has a probe probe-$\mathcal{G}$ completion under this partition, $G = (P \cup N, E)$ is called a partitioned probe-$\mathcal{G}$ graph.
- Notice that by Robert's result follows that probe unit interval graphs and probe proper interval graphs are the same class. For probe interval (unit) graphs we call the vertices in P (resp. N) probe intervals (resp. nonprobe intervals) indistinctly.

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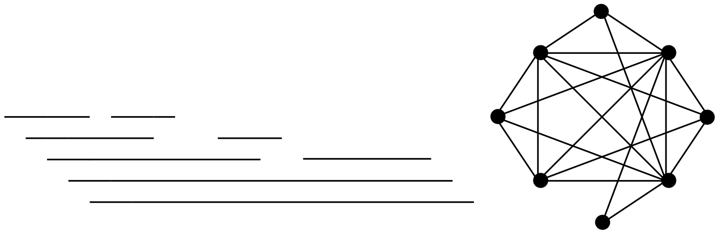

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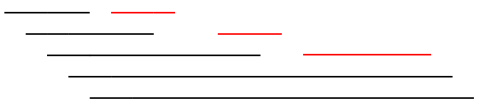

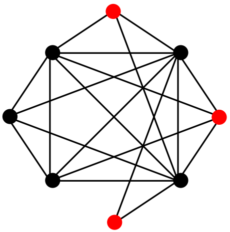

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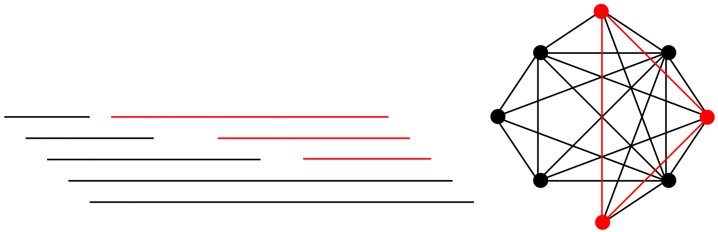

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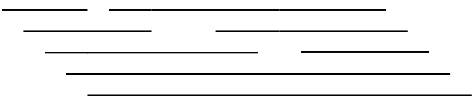
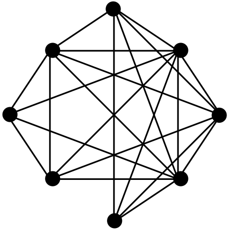
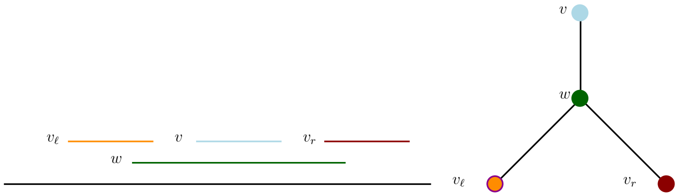




Special intervals and edges

- A vertex v of an interval graph with an interval model $\mathcal{G}$ is called a centered vertex under $\mathcal{G}$ or simply a centered vertex if there is an interval $I_w \in \mathcal{G}$ containing properly its corresponding interval $I_v \in \mathcal{G}$ and there are two vertices v_ℓ and v_r, adjacent to v, so that I_{v_ℓ} is completely to the left of I_v and I_{v_r} is completely to the right of I_v, in $\mathcal{G}$.



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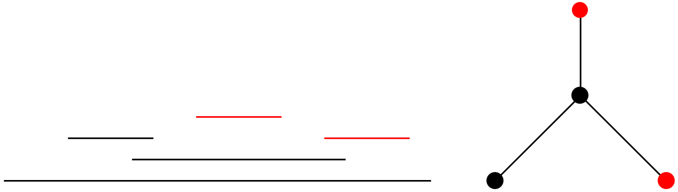

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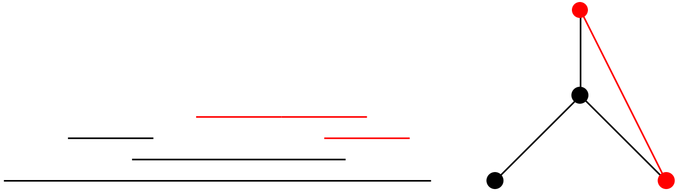

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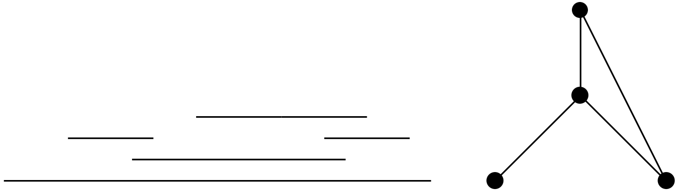
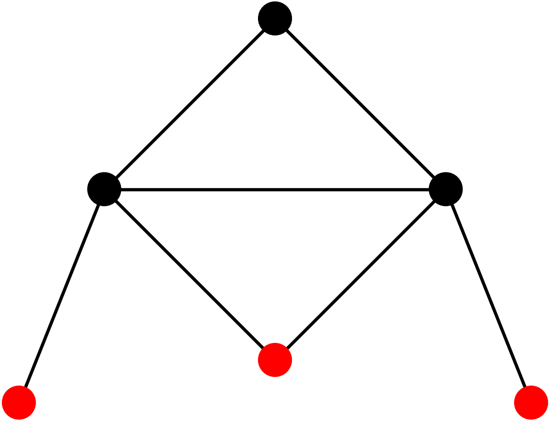


Minimal forbidden partitioned probe unit interval graph



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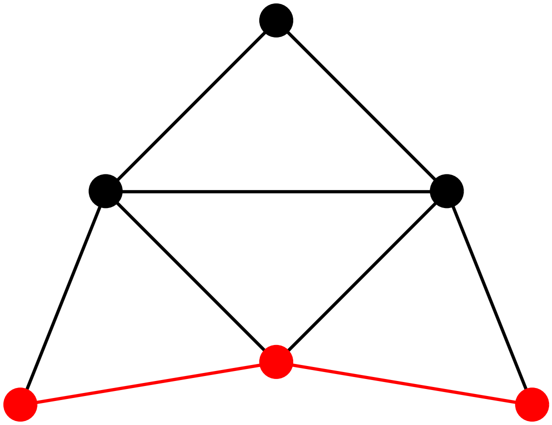

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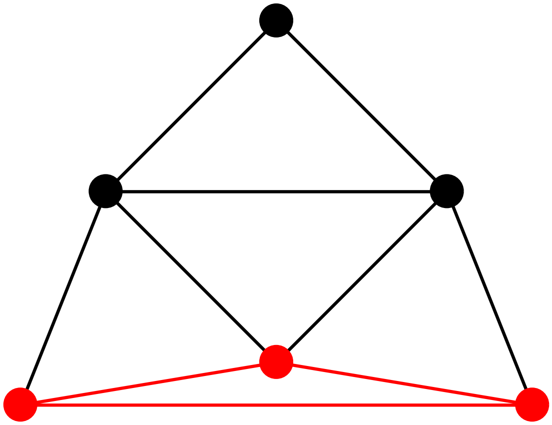

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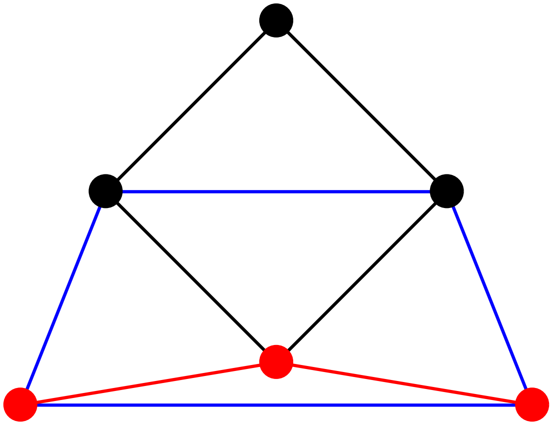
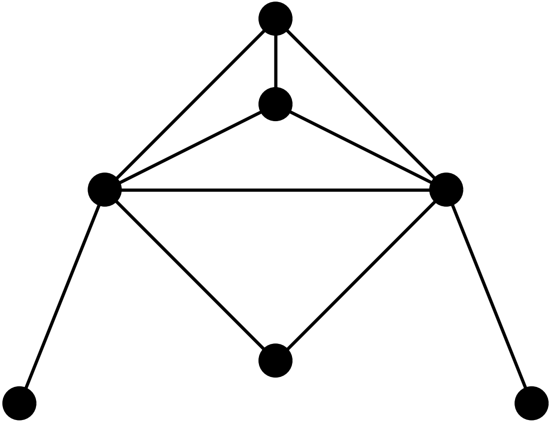


Minimal forbidden unpartitioned probe unit interval subgraphs



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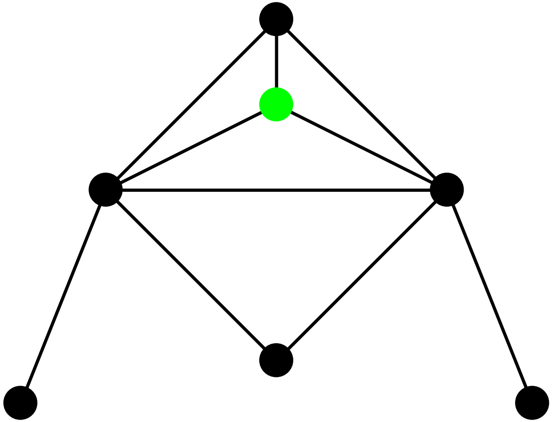

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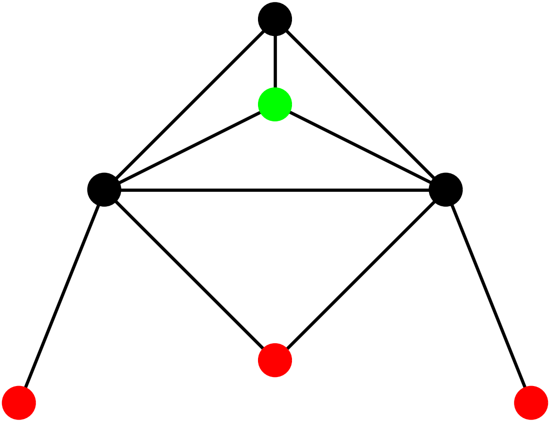

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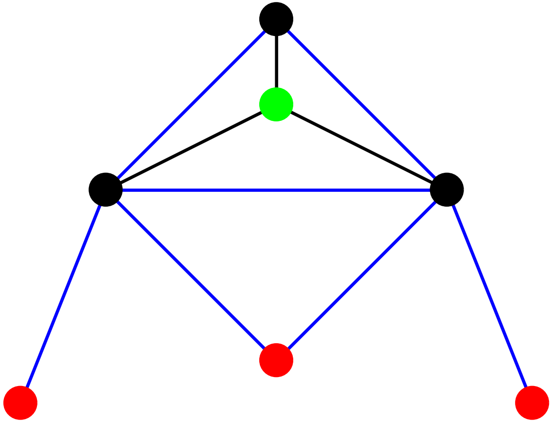

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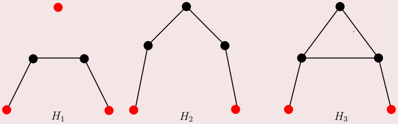
Remark

Given a graph G, then G^+ is probe unit interval if and only if G is probe-$\mathcal{R}$.

Partitioned probe-$\mathcal{R}$ interval graphs

Lemma

Given a partitioned interval graph $G = (P \cup N, E)$, then $G = (P \cup N, E)$ is a partitioned probe-$\mathcal{R}$ graph if and only if every three pairwise nonadjacent vertices has at most one vertex in P and it does not contain any of the following graph as partitioned induced subgraph.



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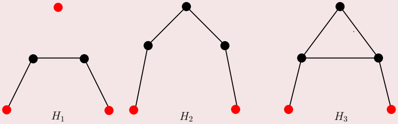

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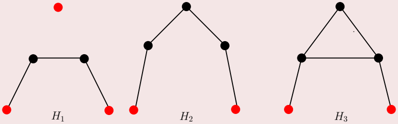

Sketch of the proof

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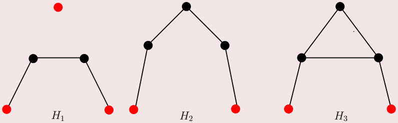
Partitioned probe-$\mathcal{R}$ interval graphs

Lemma

Given a partitioned interval graph $G = (P \cup N, E)$, then $G = (P \cup N, E)$ is a partitioned probe-$\mathcal{R}$ graph if and only if every three pairwise nonadjacent vertices has at most one vertex in P and it does not contain any of the following graph as partitioned induced subgraph.



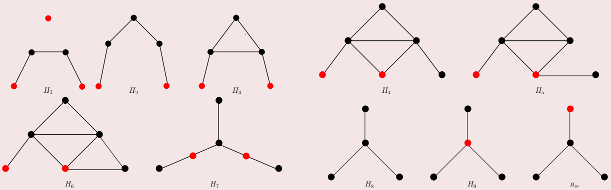
Sketch of the proof

It is easy to see that G contains no three pairwise nonadjacent vertices with more than one probe vertex and contains no graph of the figure as partitioned induced subgraph. Conversely, since G contains no three pairwise nonadjacent vertices with more of one probe vertex and contains no H_1 as partitioned induced subgraph then, if G is disconnected, every connected component of G is P_4-free and thus it can be proved that G is a partitioned probe-$\mathcal{R}$ graph. If G is connected, since G does not contain H_2 and H_3 as partitioned induced subgraph and it contains no three pairwise nonadjacent vertices with more than one probe vertex, then it can be proved that G is a probe-$\mathcal{R}$ graph. $\square$

Partitioned probe unit interval graphs

Theorem

Given a partitioned interval graph G, then G is a partitioned probe unit interval graph if and only if G contains no H_i^+ for $1 \leq i \leq 3$ and H_j for $4 \leq j \leq 10$ as partitioned induced subgraphs.



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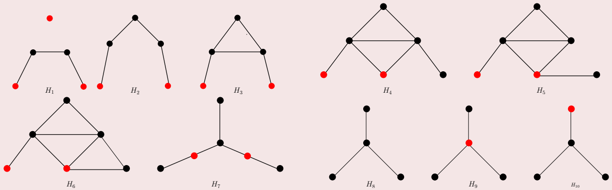

Sketch of the proof The necessary condition is easy to prove.

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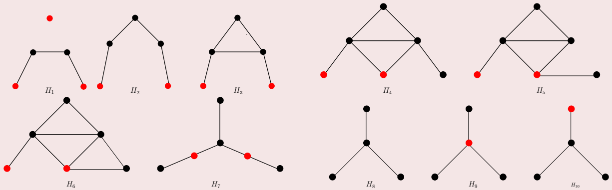

Sketch of the proof The necessary condition is easy to prove. Suppose that G is a partitioned interval graph and it contains no H_i^+ for $1 \leq i \leq 3$ and H_j for $4 \leq j \leq 10$ as partitioned induced subgraphs having an interval model $\mathcal{G}$.

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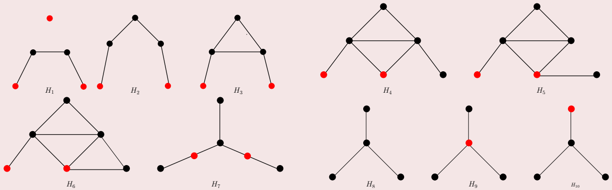

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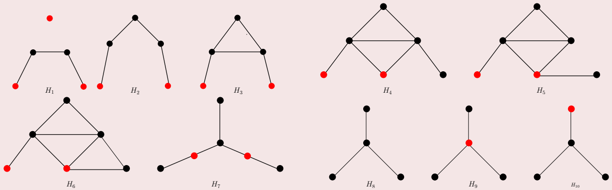

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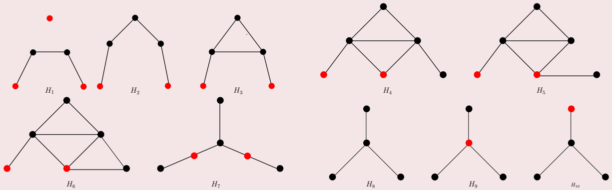

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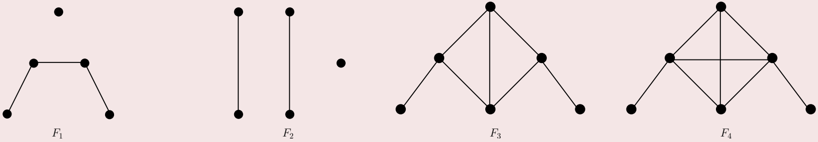
Unpartitioned probe-$\mathcal{R}$-graph

Lemma

Given an interval graph G, then G is a probe-$\mathcal{R}$ graph if and only if G does not contain any of the following graphs as forbidden induced subgraphs.



Sketch of the proof

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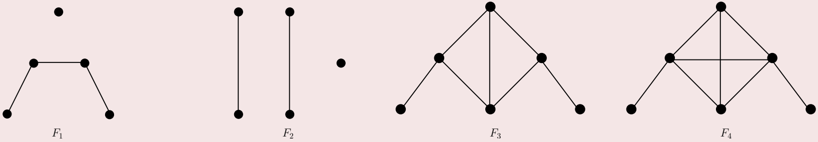

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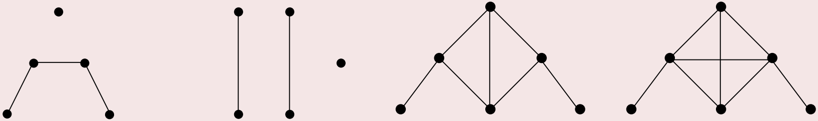

F_1 F_2 F_3 F_4

Sketch of the proof

The necessary condition is easy to prove. Conversely, if G is disconnected and does not contain F_1 and F_2 as induced subgraph, then, it can be proved that, G is a probe-$\mathcal{R}$ graph.

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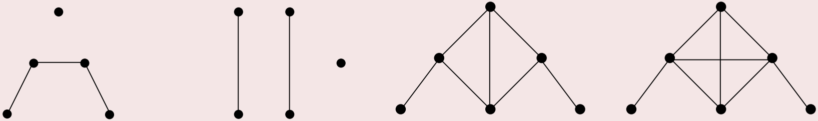


F_1 F_2 F_3 F_4

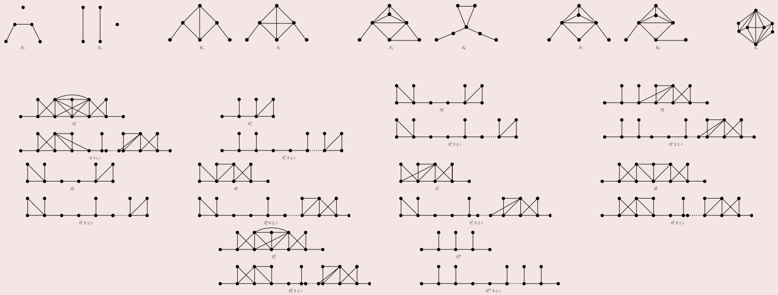
Sketch of the proof

The necessary condition is easy to prove. Conversely, if G is disconnected and does not contain F_1 and F_2 as induced subgraph, then, it can be proved that, G is a probe-$\mathcal{R}$ graph. If G is connected, the vertex set can partitioned into probe and nonprobe vertices in such a way that, since G is $\{F_1, F_2, F_3, F_4\}$-free, the resulting partitioned graph does not contain three pairwise nonadjacent vertices with at least two of them in P and H_i for $1 \leq i \leq 3$, and thus G is a probe-$\mathcal{R}$ graph. $\square$

Unpartitioned probe unit interval graphs

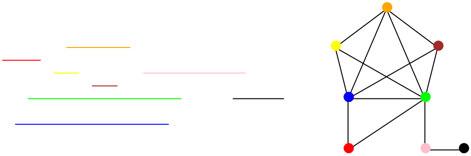
Theorem

Given an interval graph G, then G is probe unit interval if and only if it contains no F_i^+ for $1 \leq i \leq 4$, F_i for $5 \leq i \leq 9$ and S_i^j for $1 \leq i$ and $1 \leq j \leq 10$.



Sketch of the proof

We can assume that G is a connected interval graph. The necessary condition is easy to check. Conversely, if G is an interval graph with at least a universal vertex and having no F_i^+ for $1 \leq i \leq 4$ as induced subgraph, then G is probe unit interval. Suppose now that G does not have any universal vertex and it does not have any graph F_i for $i = 2$ and $5 \leq i \leq 9$ as induced subgraph, then G has an interval model $\mathcal{G}$ with no two centered intervals with a at least a point in common. Consequently, since G has such an interval model and does not have any S_i^j for $1 \leq i$ and $1 \leq j \leq 10$ as forbidden induced subgraph, it can be proved that the vertices of G can be partitioned in such a way that its corresponding partitioned interval model satisfies the condition of the characterization for the partitioned case. $\square$



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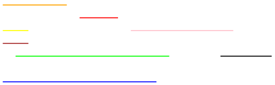

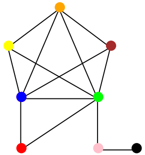

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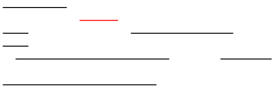

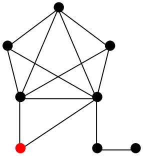

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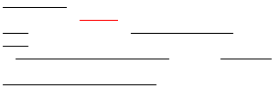

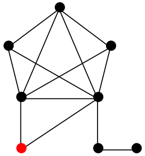

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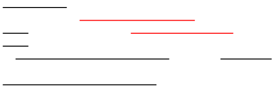

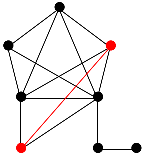

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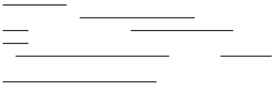

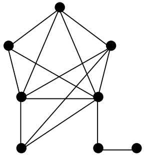

¡Muchas gracias por su atención!