On Societies Choosing Social Outcomes, and their Memberships: Strategy-proofness

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Introduction

- A classical social choice problem is the following:
  - A society N of agents has to choose an outcome from a given set X.
- Agents may have different preferences over X, and it is desirable that the chosen outcome be perceived as a compromise among the potentially different preferences, agents have to be asked about their preferences over X.
- A social choice function collects individual preferences over X and selects in a systematic way an outcome taking into account the revealed preference profile.
- This classical approach assumes that the composition of the society is independent of the chosen outcome.

Introduction

- There are many situations for which this assumption is not appropriate because the composition of the society may depend on the chosen outcome.
- For instance, membership of a political party may depend on the positions that the party takes on issues like the death penalty, abortion, or the **possibility of allowing the independence of a region of the country**.
- A professor in a department may consider to look for a position in another university if he considers that the recruitment of the department has not being satisfactory to his standards.
- To be able to deal with such situations the classical social choice model has to be modified to include explicitly the possibility that members may leave the society as the consequence of the chosen outcome.

Introduction

- Barberà, Mashler and Shalev (2001) consider a dynamic setting in which the sets of founders and candidates are fixed, and the society holds elections for a fixed number of periods using voting by quota 1.
- They show that very interesting strategic behavior may emerge in equilibrium, even when the used voting method is very simple.
- Barberà and Perea (2002) study a similar model in which the transfer of influence to new members or non elected candidates behaves in a continuous way instead of being binary. They study the (essentially) unique subgame perfect equilibrium of a model with these features and identify its simple dynamic structure.

Introduction

- In this paper:
  - We consider that the set of alternatives are all pairs formed by a subset of the original society and an outcome in X.
  - We assume that agents' preferences are defined over the set $2^N \times X$ of alternatives and satisfy two natural requirements:
    - each agent has strict preferences between any two alternatives, provided the agent belongs to the two corresponding societies.
    - each agent is indifferent between two alternatives, provided the agent is not a member of any of the two corresponding societies.

Introduction

- We consider rules that operate on this restricted domain of profiles by selecting, for each profile, an alternative (a final society and an outcome).
- An agent that understands the effect of the revealed preference on the selected alternative faces an strategic problem: how to select the best revealed preference.
- Depending on the rule under consideration, the agent may realize that the solution to this problem is ambiguous because it may depend on the agent's expectations that he has about the revealed preferences of the other agents, and in turn he may also realize that to formulate hypothesis about those revealed preferences require hypothesis about the others' expectations, and so on.

Introduction

- Strategy-proof rules make all these considerations unnecessary since truthtelling is a weakly dominant strategy of the direct revelation game form at each profile; namely, each agent's decision problem is independent of the preferences revealed by the other agents.
- Theorem 1, characterizes the class of all strategy-proof, unanimous and nonbossy rules as the family of all serial dictator rules.
  - A rule is unanimous if it always selects an alternative belonging to the set of common best alternatives, whenever this set is nonempty.
  - A rule is nonbossy if it is invariant with respect to the change of preferences of an agent who is not a member of the two final societies.
  - A serial dictator rule, relative to an ordering of the agents, gives to the first agent the power to select his best alternative, and only if this agent has many indifferent alternatives at the top of his preference then, the second agent in the order has the power to select his best alternative among those declared as being indifferent by the first agent, and proceeds similarly following the ordering of the agents.

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- Agent i considers alternative (S, x) to be at least as good as alternative (T, y).
- Let P_i and I_i denote the strict and indifference relations induced by R_i over A.

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- We assume that agent i's preferences R_i over A satisfy the following two properties: for all $x, y \in X$ and $S, T \in 2^N$,

(P.1) if $i \notin S \cup T$ then $(S, x)\, I_i\, (T, y)$; and

(P.2) if $i \in S \cap T$ and $(S, x) \neq (T, y)$ then either $(S, x)\, P_i\, (T, y)$ or $(T, y)\, P_i\, (S, x)$.

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- Let $\mathcal{R}_i$ be the set of preferences of agent $i \in N$ over A satisfying (P.1) and (P.2), and let $\mathcal{R} = \times_{i \in N} \mathcal{R}_i$ be the set of (preference) *profiles*.

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- Given R_i and (S, x) we write $(S, x) R_i [\varnothing]_i$ to represent that $(S, x) R_i (T, y)$ for all $(T, y) \in [\varnothing]_i$.

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- The *choice* of agent i in A' at R_i is:

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- The *top* of R_i,

$$\tau\left(R_i\right) = \left\{(S, x) \in A \mid (S, x)\, R_i\, (T, y) \text{ for all } (T, y) \in A\right\}.$$

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- We will often write $f(R)$ as $(f_N(R), f_X(R))$, where $f_N(R) \in 2^N$ and $f_X(R) \in X$.

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- For $\pi \in \Pi$ and $1 \leq k \leq n$, we write π_k to denote the agent $\pi^{-1}(k)$.
- Let $S \in 2^N$ be a subset of agents and π be a permutation of N. Denote

$$\pi(S) = \{i \in N \mid \pi(j) = i \text{ for some } j \in S\}$$

Properties of rules

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<u>Strategy-proofness</u> For all $R \in \mathcal{R}$, all $i \in N$ and all $R_i' \in \mathcal{R}_i$, $f(R_i, R_{-i}) R_i f(R_i', R_{-i})$.

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<u>STRATEGY-PROOFNESS</u> For all $R \in \mathcal{R}$, all $i \in N$ and all $R'_i \in \mathcal{R}_i$,
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<u>NONBOSSINESS</u> For all $R \in \mathcal{R}$, all $i \in N$ and all $R'_i \in \mathcal{R}_i$ such that
$i \notin f_N(R) \cup f_N(R'_i, R_{-i})$, $f(R'_i, R_{-i}) = f(R)$.

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- Fix a profile $R \in \mathcal{R}$
  - look for the best alternative (S_1, x_1) of agent π_1, the first in the ordering induced by π. If $\pi_1 \in S_1$, set $f^{\pi,x}(R) = (S_1, x_1)$.

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  - Otherwise, look for the best alternative (S_2, x_2) of agent π_2, the second in the ordering induced by π, with the property that $\pi_1 \notin S_2$. If $\pi_2 \in S_2$, set $f^{\pi,x}(R) = (S_2, x_2)$.

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    - Otherwise, look for the best alternative (S_3, x_3) of agent π_3, the third in the ordering induced by π, provided that $\pi_1, \pi_2 \notin S_3$, and so on.

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  - Otherwise, look for the best alternative (S_3, x_3) of agent π_3, the third in the ordering induced by π, provided that $\pi_1, \pi_2 \notin S_3$, and so on.
  - At the end, look for the best alternative (S_n, x_n) of agent π_n, the last in the ordering induced by π, with the property that for each $i \in \{1, ..., n-1\}$, $\pi_i \notin S_n$. If $\pi_n \in S_n$, set $f^{\pi,x}(R) = (S_n, x_n)$.

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  - At the end, look for the best alternative (S_n, x_n) of agent π_n, the last in the ordering induced by π, with the property that for each $i \in \{1, ..., n-1\}$, $\pi_i \notin S_n$. If $\pi_n \in S_n$, set $f^{\pi,x}(R) = (S_n, x_n)$.
  - Otherwise, and since no agent wants to stay in the society whatever element of X is selected, set $f^{\pi,x}(R) = (\varnothing, x)$. So, x plays the role of the residual outcome only when no agent wants to stay in the society under any circumstance.

Strategy-proof rules

- Formally. Fix $\pi \in \Pi$ and $x \in X$. Let $R \in \mathcal{R}$ be a profile. Define $f^{\pi,x}(R)$ recursively, as follows.

Stage 1. Let $A_1 = A$. Consider two cases:

1. $|C(A_1, R_{\pi_1})| = 1$. Then, $C(A_1, R_{\pi_1}) = \tau(R_{\pi_1})$. Set $(S_1, x_1) = C(A_1, R_{\pi_1})$ and observe that $\pi_1 \in S_1$. Define

$$f^{\pi,x}(R) = (S_1, x_1).$$

2. $|C(A_1, R_{\pi_1})| > 1$. Then, $C(A_1, R_{\pi_1}) = \{(S, x') \in A \mid \pi_1 \notin S \text{ and } x' \in X\} = [\varnothing]_{\pi_1}$. Go to Stage 2.

We now define Stage k $(1 < k < n)$, assuming that the stage $k-1$ has been reached and A_{k-1} was defined on it.

Stage k. Let $A_k = C(A_{k-1}, R_{\pi_{k-1}})$. Consider two cases.

1. $|C(A_k, R_{\pi_k})| = 1$. Then, $C(A_k, R_{\pi_k}) = (S_k, x_k)$ and $\pi_k \in S_k$. Define
$$f^{\pi,x}(R) = (S_k, x_k).$$
2. $|C(A_k, R_{\pi_k})| > 1$. Then, $C(A_k, R_{\pi_k}) = \{(S, x') \in A \mid \pi_i \notin S \text{ for all } i \leq k \text{ and } x' \in X\} = \bigcap_{j=1}^{k-1} [\varnothing]_{\pi_j}$. Go to Stage $k+1$.

Strategy-proof rules

We now define Stage n, the last stage of the procedure, assuming that the stage $n-1$ has been reached and A_{n-1} was defined on it.

Stage n. Let $A_n = C(A_{n-1}, R_{\pi_{n-1}})$.

1. $|C(A_n, R_{\pi_n})| = 1$. Then, $C(A_n, R_{\pi_n}) = (S_n, x_n)$ and $\pi_n \in S_n$. Define
$$f^{\pi,x}(R) = (S_n, x_n).$$
2. $|C(A_n, R_{\pi_n})| > 1$. Then,
$C(A_n, R_{\pi_n}) = \{(\varnothing, x') \in A \mid x' \in X\} = \bigcap_{j=1}^{n} [\varnothing]_{\pi_j}$. Define
$$f^{\pi,x}(R) = (\varnothing, x).$$

Strategy-proof rules

Example 1 Let $N = \{1, 2\}$ be the set of agents, $X = \{a, b, c\}$ be the set of outcomes and consider the identity permutation $\pi = (\pi_1, \pi_2) = (1, 2)$ and $x = a$. We apply the serial dictator rule $f^{(1,2),a}$ to the following preferences,

R_1	R_1'	R_2	R_2'
(N, b)	$\{(S, y) \in A \mid 1 \notin S\}$	(N, a)	(N, a)
		(N, b)	(N, b)
		$(2, c)$	$\{(S, y) \in A \mid 2 \notin S\}$

.

Then, $f^{(1,2),a}(R_1, R_2) = (N, b)$, $f^{(1,2),a}(R_1, R_2') = (N, b)$, $f^{(1,2),a}(R_1', R_2) = (\{2\}, c)$, and $f^{(1,2),a}(R_1', R_2') = (\varnothing, a)$.

Strategy-proof rules

Theorem 1 *Assume* $|X| \geq 3$. *A rule* $f : \mathcal{R} \to A$ *is strategy-proof, unanimous and nonbossy if and only if* f *is a serial dictator rule for some permutation* $\pi \in \Pi$ *and alternative* $x \in X$.

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