Asking Infinite Voters 'Who is a J?´ and some Group Identification Problem Topics

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Reunión Anual de la UMA - Septiembre 2016

Who is a J? (Kasher and Rubinstein - 1997)

- They use a Social Choice approach to deal with the problem of determining who is a J in a society
- Use aggregators that take in count the opinion of each member of the society
- Characterize three Collective Identity Functions (CIF) using diferent sets of axioms

- *The Liberal Aggregator*: an individual is a J if and only if one defines oneself to be a J
- *The Dictatorship Aggregator*: a pre-designated member of the given society determines who is a J
- *The Oligarchical Aggregator*: two members of the given society belong to the same group if and only if they are both considered to have the same collective identity by all members of a pre-designated subgroup in the society

Goals

- **Infinite Case**: find if the results of K-R still hold when the number of voters is infinite
- **Liberal Case**: working with different notions of Liberalism, see if we can find some possibility results

Infinite Voters - Liberal Case

- N individuals
- Each individual i has an "opinion" described by a set $J_i \subseteq N$ of individuals that i thinks belong to class J
- J a CIF taking a profile of opinions $(J_1, \ldots, J_N)$ and yielding a set $J(J_1, \ldots, J_N) \subseteq N$

Axiomatic

- **Monotonicity (MON)**
- **Independence (I)**
- **Consensus (C)**
- **Symmetry (SYM)**
- **Liberal Principle (L)**

Strong Liberal CIF

K-R introduced a particular CIF, defined as

$$J = \{i \mid i \in J_i\}$$

that is, are J the individuals that think that are J

Results

K-R proved that

Theorem

The Strong Liberal CIF is the only CIF that verifies (C), (I), (MON), (SYM) and (L) when $\mid N \mid < \infty$.

Sung and Dimitrov (2003) proved that (C) and (MON) can be derived from the other axioms. So using a transfinite induction argument in their demonstration we can show that:

Theorem

The Strong Liberal CIF is the only CIF that verifies (I), (SYM) and (L), even when the number of voters is infinite.

Infinite Voters - Oligarchic Case

- N individuals
- Each $i \in N$ specifies an equivalence relation on N denoted $\sim_i$
- A CIF* is a function that assigns to each profile $(\sim_1, \ldots, \sim_i, \ldots)$ an equivalence relation $\sim$ over N

Axiomatic

- **Independence (I^*)**
- **Consensus (C^*)**

Oligarchic CIF*

If there exists a non-empty subset M verifying that $i \sim j$ if and only if $i \sim_k j$ for all $k \in M$ the CIF* is called Oligarchic

Results

Barthelemy, Leclerc and Monjardet (1986) characterize the oligarchic aggregators when N is finite:

Theorem

The only CIF* that satisfies (C^*) and (I^*) are oligarchic

When the number of individuals is infinite, this result does no longer hold. We have instead (using a weakly anti-mainstream CIF*):

Theorem

If $N = \mathbb{N}$, the oligarchic CIF* is not the only CIF* that satisfies (C^*) and (I^*)

Finite Voters - Liberal Case

K-R introduced the following definition:

- **The Liberal Principle**(L): if there is an i such that $i \in J_i$, then $J \neq \emptyset$, and if there is an i such that $i \notin J_i$, then $J \neq N$

We adapt to this framework the different versions of liberalism that Sen used (1970):

- **Liberalism**(SL): for each $i \in N$, there exists a $j \in N$ such that if $j \in J_i$, then $j \in J$; and if $j \notin J_i$, then $j \notin J$
- **Minimal Liberalism**(ML): there exists at least an $i, j \in N$, $i \neq j$, and $k, l \in N$, $k \neq l$, such that if $k \in J_i$ then $k \in J$, if $l \in J_j$ then $l \in J$, if $k \notin J_i$ then $k \notin J$ and if $l \notin J_j$ then $l \notin J$.

Finite Voters - Liberal Case (cont.)

- **Super Minimal Liberalism**(SML): there exists at least $i, j \in N$, $i \neq j$, and $k, l \in N$, $k \neq l$, such that if $k \in J_i$ then $k \in J$ or if if $k \notin J_i$ then $k \notin J$ and if $l \in J_j$ then $l \in J$ or if $l \notin J_j$ then $l \notin J$

Now we introduce a new concept of liberalism, an extreme one:

- **Extreme Liberalism**(EL) is characterized by:
  - (i) If there are $i, j \in N$ such that $j \in J_i$, then $J \neq \emptyset$.
  - (ii) if there are $i, j \in N$ such that $j \notin J_i$, then $J \neq N$.

Results

Theorem

The Strong Liberal CIF is not the only CIF that verifies (C), (MON), (I), (SYM) and (SML)

For example, the Unanimity CIF verifies the five properties.

Theorem

The Strong Liberal CIF is the only CIF that verifies (C),(MON), (SYM), (I) and (SL).

Corollary

The Strong Liberal CIF is the only CIF that verifies (C),(MON), (SYM), (I) and (ML).

Results (cont.)

We use the following two lemmas

Lemma

The only CIF that verifies (C), (SYM), (I), (MON) and part (i) of (EL) is the "whatever" CIF

and

Lemma

The only CIF that satisfies (MON), (C), (SYM), (I) and part (ii) of (EL) is the Unanimity CIF

to prove the next result:

Results (cont.)

Theorem

There is no CIF that verifies (C), (MON), (SYM), (I) and (EL)

Conclusions

- When we work with infinite voters, the liberal case result holds, but the oligarchic no.
- Varying the idea of liberalism, gives us possibility as well impossibility results

Future Work

The dictatorial case in the infinite case, in which the range of the function cannot be the entire society nor the empty set in the K-R framework.