

Topics in Biomathematics

A dynamical systems approach to the study of chemical, biochemical and neural processes.

Horacio G. Rotstein

1. Biochemical reactions. The law of mass action. Models of simple chemical reactions. One enzyme - one substrate chemical reaction. Enzyme catalysis and enzyme inhibition. Cooperativity. Approximate solution by singular perturbation methods for ordinary differential equations: Michaelis-Menten kinetics. Glycolysis and glycolytic oscillators.

2. Neuronal excitability. The Hodgkin-Huxley model of action potential generation in single neurons. Reduced artificially spiking neural models: integrate and fire, resonate and fire. Reduced two-dimensional relaxation oscillator models: FitzHugh-Nagumo and Morris-Lecar. Phase space analysis of neuronal excitability. The canard phenomenon (sudden explosion of limit cycles) and geometric singular perturbation theory. Neural model displaying subthreshold oscillatory activity.

3. Intercellular communication in networks of neurons. Chemical synapses. Excitatory (AMPA and NMDA) and inhibitory ($GABA_A$ and $GABA_B$) synaptic models. Gap junctions (electrical diffusion). Synchronization properties of neural networks. Genesis of rhythmic activity in the brain.

Basic Bibliography

- [1] J. Keener and J. Sneyd. *Mathematical Physiology*. Springer-Verlag.
- [2] C. C. Lin and L. A. Segel. *Mathematics Applied to Deterministic Problems in the Natural Sciences*. Society for Industrial and Applied Mathematics (SIAM), 1988.
- [3] J. D. Murray. *Mathematical Biology*. Springer, Berlin, 1989.
- [4] S. H. Strogatz. *Nonlinear Dynamics and Chaos*. Addison Wesley, Reading MA, 1994.

Additional Bibliography

- [5] A. Beuter, L. Glass, M. C. Mackey and M. S. Titcombe (eds.). *Nonlinear Dynamics in Physiology and Medicine*. Springer-Verlag, 2003.
 - [6] J. M. Bower and H. Bolouri (eds.). *Computational Modeling of Genetic and Biochemical Networks*. The MIT Press, 2001.
 - [7] P. Dayan and L. F. Abbott. *Theoretical Neuroscience*.
 - [8] I. R. Epstein and J. A. Pojman. *An Introduction to Nonlinear Chemical Dynamics: Oscillations, Waves, Patterns, and Chaos*. Oxford University Press, 1998.
 - [9] E. M. Izhikevich E.M. *Dynamical Systems in Neuroscience: The Geometry of Excitability and Bursting*. The MIT press (to be published in 2006).
- <http://www.nsi.edu/users/izhikevich/publications/index.htm> (December 2005 version).
- [10] C. Koch. *Biophysics of Computation*. Oxford University Press, 1999. The MIT Press, 2001.
 - [11] A. Scott. *Neuroscience: A mathematical primer*. Springer-Verlag, 2002.