# CHARACTERIZING AFFINE HYPERSURFACES WITH PARALLEL DIFFERENCE TENSOR

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**Abstract**: The problem of classifying hypersurfaces with affine normal parallel Difference Tensor was started by F. Dillen and L. Vrancken in their interesting paper [1] :The first result they prove reads as follows:

Theorem (A). Let $M^n$ be an affine hypersurface in $\mathbb{R}^{n+1}$ with parallel difference tensor, $\nabla K = 0$. If $K$ is not equal to zero at some point, then $M^n$ is an improper affine hypersphere whose affine metric is flat. Further, there exists a number $m : 2 \le m \le n$, such that $K^{m+1}$ is different from zero and $K^m = 0$. Besides, $M^n$ is given as the graph of a polynomial of degree $m+1$ with constant Hessian.

There are other, related results which are proven in the paper and, by using all this, they study diverse instances of the integer values for $n$ and $m$, to obtain a partial classification of some cases, particularly in lower dimensions $n = 2,3,4$. Nevertheless, it should be said, first of all, that the geometrical and analytical properties described in the above theorem are not characterizing. In fact, consider the hypersurface described by the graph immersion $f(t_1,t_2,t_3) = t_1 t_2 + t_1^2 t_3 + t_3^2$.

Then, it is easy to see that this satisfies: $\nabla K \ne 0, K^2 \ne 0, K^3 = 0$.

Furthermore, and as a consequence of this fact, the classification obtained in the mentioned paper is not complete, at least for dimensions 3 and 4.

The object of this presentation is to discuss characterizing geometrical and analytical properties for the problem considered. The method of work to be used here has already been introduced by one of us in previous papers, where the classification of affine hypersurfaces with parallel Second Fundamental (Cubic) was obtained [2, 3, 4]. By using this method we shall see that, in fact, the classification for the present class shall depend not only on the integer value $m$, as described above, but also on two other integer values that we shall label as $k$ and $r$, with $1 \le k \le n/2$, $1 \le r \le n-1$.

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