THEORY OF AFFINE SHELLS: THE USE OF NON LINEAR P.D.E. METHODS

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Abstract.

The classical Monge-Ampère equation has been the center of considerable interest in recent years because of its important role in several areas of applied mathematics. In reflecting these developments, this works stresses the geometric aspects of this beautiful theory, using some techniques from harmonic analysis---covering lemmas and set decompositions. Moreover, Monge-Ampère type equations have applications in the areas of differential geometry, the calculus of variations, and several optimization problems, such as the Monge-Kantorovitch mass transfer problem, and now in the theory of Affine Shells.

In mathematics, a (real) Monge–Ampère equation is a nonlinear second order partial differential equation of special kind. A second order equation for the unknown function u of two variables *x*,*y* is of Monge–Ampère type if it is linear in the determinant of the Hessian matrix of u and in the second order partial derivatives of *u*. The independent variables (*x*,*y*) vary over a given domain D of $\mathbb{R}^2$. The term also applies to analogous equations with n independent variables. The most complete results so far have been obtained when the equation is elliptic. Monge–Ampère equations frequently arise in differential geometry, for example, in the Weyl and Minkowski problems in differential geometry of surfaces. The equation is named after Gaspard Monge and André-Marie Ampère.

An affine shell, as we have defined it in previous articles [1, 2, 3], made of a perfectly elastic, homogeneous and isotropic material, is subjected to forces acting along its edge, thus passing from an original “unstrained” state to a final “strained” one. Our objective in this work is to approximate the geometric objects in order to asses how much the shell has been deformed. For achieving that goal we use Non Linear P.D.E. Methods. In particular, equations of Monge–Ampere type are considered. Besides, we are interested in knowing how it works the relationship with classical shells, by comparing the same case with that treated previously within the euclidean geometry context, already considered by other authors [4, 5, 6]. In order to perform the job we will use, systematically, the well known unimodular affine geometric invariants, as a frame of reference for these estimates.

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