ON MANY-VALUED MODAL LOGICS OVER FINITE RESIDUATED LATTICES

Felix Bou, Francesc Esteva, Lluis Godo[1] and Ricardo Rodriguez[2]

In this paper we consider finitely-valued modal logics with Kripke style semantics where both propositions and the accessibility relation are finitely-valued over finite BL-algebras (including constants, for every element in the algebra, in the formal language). In particular, we only consider logics with a necessity operator (the possibility one is perhaps missing). This necessity operator is defined using Fitting's definition which takes into account the fact that the accessibility relation is finitely-valued. We note that in general this operator does not satisfy the K axiom.

As a methodological step, given a finite BL-algebra we start by considering a multi-modal logic where the previous necessity operator is replaced with a family (parametrized by alpha, where alpha is a truth value different from zero) of necessity operators each one semantically defined using the classical accessibility relation given by the alpha-cut of the finitely-valued original accessibility relation. This multi-modal logic is simpler than the other one because K axiom holds for each necessity in the parametrized family. For this logic, we have been able to find a complete axiomatization. Inside this logic, we show that the original necessity operator is definable. Nevertheless in general we do not prove that the interdefinability fails.

Next, for the particular case of finite Lukasiewicz algebras we show a finite complete axiomatization for the case of the original necessity operator. The proof is based on the fact that in this case the parametrized family is also definable using the original necessity operator.

[1] IIIA
CSIC
Bellaterra, España

[2] Departamento de Computación
FCEyN-UBA
Buenos Aires, Argentina