

Spectral duality for finitely generated nilpotent minimum algebras

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based on a joint work with S. Aguzzoli and V. Marra.

In nilpotent minimum logic, the conjunction is semantically interpreted by a left-continuous (but not continuous) triangular norm; implication is obtained through residuation. We shall focus attention on finitely axiomatized theories in nilpotent minimum logic; their algebraic counterparts are finite NM-algebras. Building on results in [1] and [2], we establish a spectral (or Stone-type) duality for finite NM-algebras. We describe the dual of a finite NM-algebra as a forest (i.e., a finite partially ordered set such that below any element there lies a subset that inherits a total order), such that each one of its trees (i.e., partially ordered subsets with a minimum) is enriched by one additional bit of information. Homomorphisms of NM-algebras dualise to order-preserving maps (between the corresponding forests) satisfying appropriate additional conditions. This seems to be the first instance of a (finite) spectral duality for a logic that is based on a discontinuous t-norm. We then show that the construction is actually useful in obtaining further results: an explicit description of finite coproducts of finite NM-algebras; a strong form of amalgamation for finite NM-algebras, along with the strongest possible form of the Deduction Theorem for nilpotent minimum logic; a functional representation of free finitely generated NM-algebras; an exact recursive formula for the cardinality of free finitely generated NM-algebras.

References

- [1] M. Busaniche. Free Nilpotent Minimum Algebras, *Mathematical Logic Quarterly*, **52**, 219-236 (2006).
- [2] O. M. D'Antona and V. Marra. Computing coproducts of finitely presented Gödel algebras, *Annals of Pure and Applied Logic*, **142**, 202-211 (2005).

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