Statistical Information Theory and Geometry for SAR Image Analysis

Recent results and research topics

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Aims

- Connecting SAR (*Synthetic Aperture Radar*) image processing and analysis with statistical models
- Commenting upon seemingly disjoint problems
- Showing that they can be tackled successfully within the same framework
- Discussing the use of Information-Theoretic Statistically-Based tools

SUMMARY

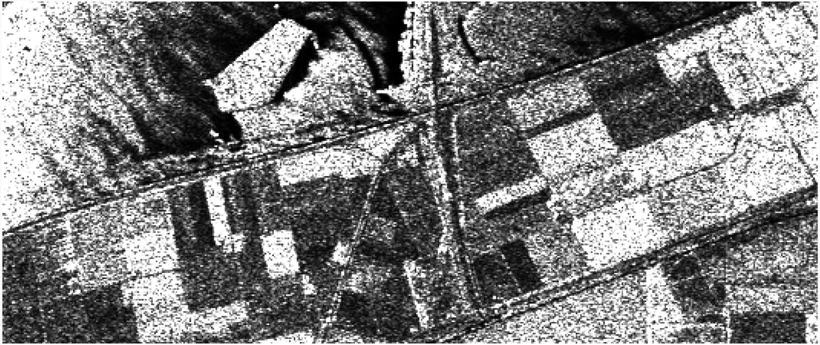
Outline

What is SAR?

Basic Characteristics

- Synthetic Aperture Radar (SAR) and Polarimetric SAR (PolSAR) sensors have been successfully used in remote sensing.
    - data can be captured independently of weather conditions, because the sensor is active,
    - they measure mostly geometry and dielectric constant, as they work in the microwaves region of the spectrum,
    - depending on the sensor and target characteristics, the signal penetrates through soil, canopies etc.
- SAR and PolSAR systems can provide images with high spacial resolution but contaminated by an interference pattern, called speckle.

SAR Image



General Discussion

- ☞ Speckle imposes a peculiar behavior to the SAR return:
  - a good model for the return is multiplicative, rather than additive, and
  - intensities follow non-Gaussian distributions.
- ☞ This makes SAR image analysis challenging because standard methods for image processing
  1. assume additivity $Z = X + Y$, and
  2. use tools derived from the Gaussian assumption.
- ☞ Observations are organized as a complex-valued Hermitian positive definite matrix in each position of a fully PolSAR image.
- Thus specialized tools are required.

Outline

A well accepted model for the return Z in each pixel is the Multiplicative Model:

$$Z = XY,$$

where X is the backscatter, and Y is the speckle.

The physics of the imaging allows to assume that Y follows a Gamma distribution with unitary mean and shape parameter $L \geq 1$, the number of looks. We denote this $Y \sim \Gamma(1, L)$.

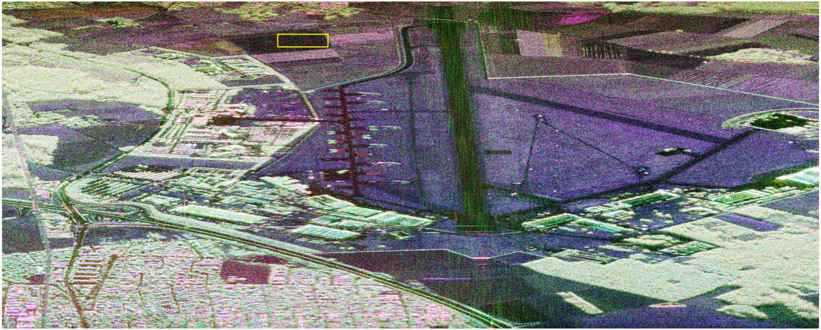
We are interested in the backscatter X, which can be constant or random but always positive.

Is this model correct?

Box et al. (2005)
The most that can be expected from any model is that it can supply an useful approximation to reality:

All models are wrong; some models are useful.

Is this model correct?



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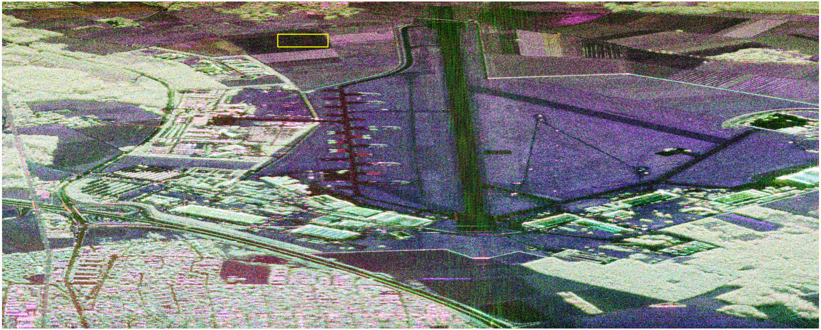

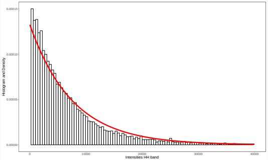

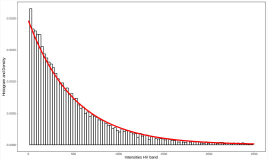

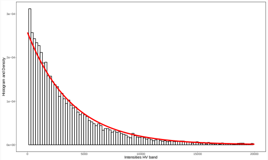

Outline

Assessment of Despeckling Filters

There are many despeckling techniques, as well as measures of their performance.

Assuming the multiplicative model, the observed image Z is the product of two independent fields: the backscatter X and the speckle Y.

The result of any speckle filter is $\widehat{X}$, an estimator of the backscatter X, based solely on the observed data Z.

An ideal estimator would be the one for which the ratio $I = Z/\widehat{X}$ is only speckle: a collection of independent identically distributed samples from Gamma variates.

We, then, assess the quality of a filter by the closeness of the ratio image I to the hypothesis that it is adherent to the statistical properties of pure speckle.

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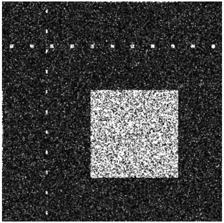
PROPOSAL

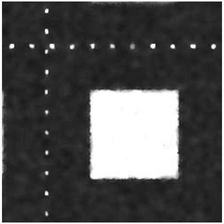
We propose an evaluation of the quality of the remaining speckle based on two components:

First-order component: one term for mean preservation, and another for preservation of the equivalent number of looks.

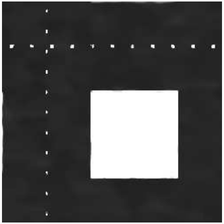
Second-order component: a measure of the remaining geometrical content within the ratio image.



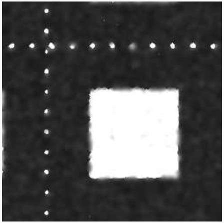
Noisy



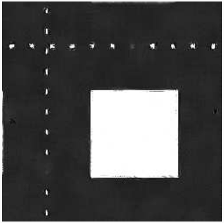
Lee



FANS



SRAD



PPB

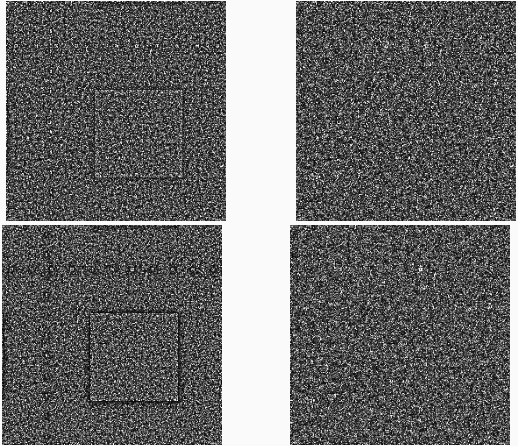


Figure 1: Top: E-Lee and FANS. Bottom: SRAD and PPB.

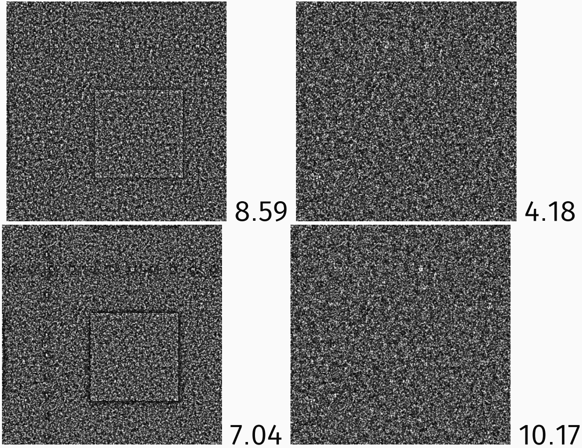


Figure 1: Top: E-Lee and FANS. Bottom: SRAD and PPB.

The Rationale

The rationale behind this proposal is measuring the distance between two objects, namely

- a theoretical distribution, and
- observed data.

This was done in an empirical manner, and provided very good results; cf. Gomez et al. (2017*b*, 2019).

A General Approach

This idea of comparing samples, or samples and models, has led to interesting tools.

First, we will see a model with respect to which such comparisons will be made. Then we will see measures based on Information Theory and Information Geometry. Finally, we will see a multivariate model for PolSAR data, and several applications of these techniques.

Outline

The $\mathcal{G}^0$ Return Model

We call $Y \sim \Gamma(1, L)$ speckle, and $X = \sigma^2$ backscatter, and we assume these random variables are independent.

The return is $Z = XY$.

When the mean backscatter σ^2 fluctuates, the return $\sigma^2 Y$ is no longer Gamma-distributed.

The two most important stochastic models for the backscatter are:

- A Gamma random variable
- A Reciprocal Gamma random variable

The $\mathcal{G}^0$ Return Model

Assuming $X \sim \Gamma^{-1}(\alpha, \gamma)$ and $Y \sim \Gamma(1, L)$, one obtains the $\mathcal{G}^0$ distribution for the return $Z = XY$, which is characterized by the density

$$f_Z(z; \alpha, \gamma, L) = \frac{L^L \Gamma(L - \alpha)}{\gamma^\alpha \Gamma(L)\Gamma(-\alpha)} \frac{z^{L-1}}{(\gamma + Lz)^{L-\alpha}}, \qquad (1)$$

where $\alpha < 0$, and $\gamma, z > 0$.

The parameters describe the texture (the smaller the value of α is, the less textured the region is), and the scale (γ).

This model was proposed by Frery et al. (1997).

$\mathcal{G}^0$ DENSITIES

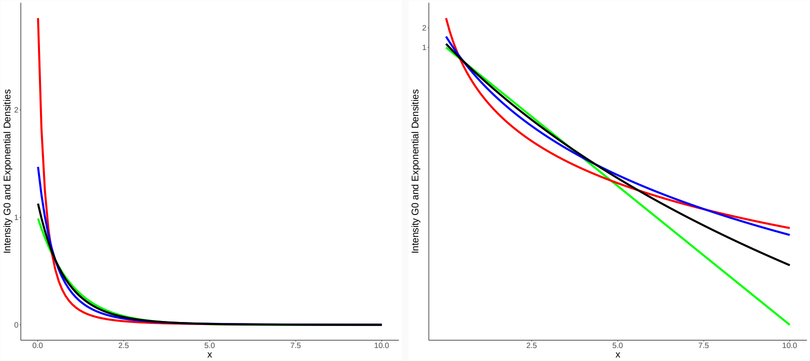


Figure 2: Densities in linear and semilogarithmic scale of the E(1) (green) and $\mathcal{G}^0$ distributions with unitary mean and $\alpha \in \{-8, -3, -1.5\}$ in black, blue and red, resp.

Advantages and disadvantages

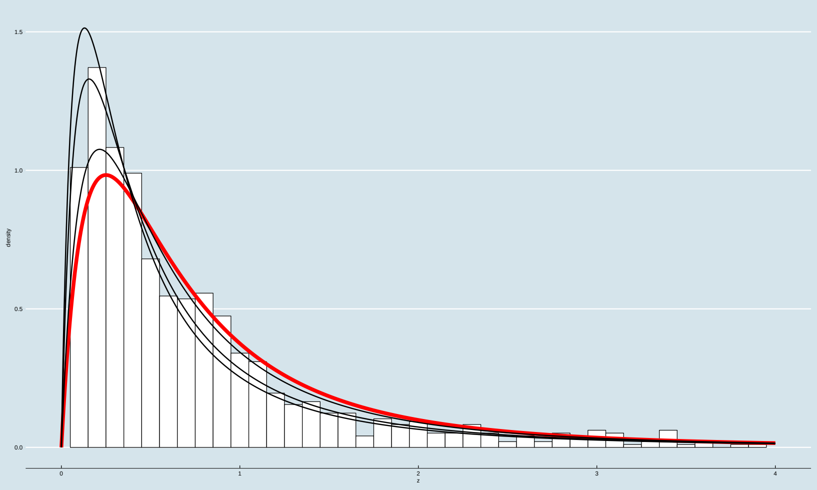
The Good

- More flexible than the Γ model
- It has the Γ model as particular case
- It relates to the Fisher-Snedekor distribution
- Differently from the $\mathcal{K}$ law, it does not involve Bessel functions

The Bad

- Its likelihood function may become flat
- Its moments estimators may have no solution
- Those estimators are susceptible to contamination

Parameter Estimation



Parameter Estimation

Gambini et al. (2015) proposed estimating by searching in Θ for the point $\widehat{\theta}$ which minimizes a distance between the evidence (data) and the model:

$$\widehat{\theta} = \arg\min_{\theta\in\Theta} d\Big(\widetilde{H}(\boldsymbol{z}), f(z;\theta)\Big),$$

where $\boldsymbol{z}$ are the data, and $\widetilde{H}$ is an smoothed version of the histogram. Also,

$$d(f_1, f_2) = \int \frac{(f_1 - f_2)^2}{f_1 + f_2},$$

is the Triangular (or symmetric χ^2) distance.

It works quite well!

Why such a distance?

We chose the Triangular distance, because of its numerical simplicity and stability, within a family of divergences between members of the same law:

h-ϕ divergences

$$D^h_\phi(\theta_1, \theta_2) = h\Big(\int \phi\Big(\frac{f(\theta_1)}{f(\theta_2)}\Big) f(\theta_2)\Big),$$

where h is strictly increasing with $h(0) = 0$, and ϕ is convex with mild smoothness properties. Then,

h-ϕ distances

$$d^h_\phi(\theta_1, \theta_2) = \frac{1}{2}\big(D^h_\phi(\theta_1, \theta_2) + D^h_\phi(\theta_2, \theta_1)\big).$$

Suitable choices of h and ϕ lead to, among others, the Bhattacharya, Rényi, Triangular, Harmonic, χ^2, Kullback-Leibler, Hellinger, Havrda-Charvát, and Sharma-Mittal distances.

Another family of distances

Consider the model $\mathcal{D}(\theta)$, two parameters $\theta^1, \theta^2 \in \Theta$, and let t be the parameter of a curve $\theta(t) \in \Theta$ which joins $\theta^1 = \theta(t_1)$ and $\theta^2 = \theta(t_2)$.

The (Shannon) geodesic distance between the models is given by:

$$s(\theta^1, \theta^2) = \left| \int_{t_1}^{t_2} \sqrt{\sum_{i,j=1}^{r} -E\left(\frac{\partial^2}{\partial\theta_i \partial\theta_j} \ln f(z \mid \theta)\right) \frac{d\theta_i}{dt} \frac{d\theta_j}{dt}} dt \right|. \qquad (2)$$

This expression can be connected with the entropy of $\mathcal{D}(\theta)$, and with generalized entropies:

$$H_\phi^h(\theta) = h\left(\int \phi(f) f \right),$$

leading to h-ϕ geodesic distances.

This is tough to obtain in most cases.

What do we have so far?

- It seems that the distance between the data and a model produces interesting results:
  - measuring it quantifies the quality of filters;
  - minimizing it yields robust estimators.
- But… is there a notion of close and far? Not yet.
- Can we compare such distances? No!

When indexed by MLE, under $H_0 : \theta_1 = \theta_2$ holds that

$$\frac{N_1 N_2}{(N_1 + N_2) h'(0) \phi''(1)} d_\phi^h(\widehat{\theta}_1, \widehat{\theta}_2)$$

follows asymptotically a χ^2_p distribution, where

- p is the dimension of θ_i,
- N_1 and N_2 are the sample sizes used to compute the MLEs $\widehat{\theta}_1$ and $\widehat{\theta}_2$,
- $N_1, N_2 \to \infty$ at comparable rate.

For the sake of simplicity, but without loss of generality, assume we have two samples of the same size N. When indexed by MLE, under $H_0 : \theta_1 = \theta_2$ holds that

$$N \frac{\left(H(\widehat{\theta}_1) - H(\widehat{\theta}_2)\right)^2}{\sigma^2(\widehat{\theta}_1) + \sigma^2(\widehat{\theta}_2)}$$

follows asymptotically a χ^2_{p-1} distribution, where p is the dimension of θ_i, and $N \to \infty$.

This can be generalized to many samples of different sizes.

What do we have now?

Test Statistics based on IT

- Each h-ϕ divergence can be turned into a test statistic.
- Each h-ϕ entropy can be turned into a test statistic.

Test Statistics based on IG

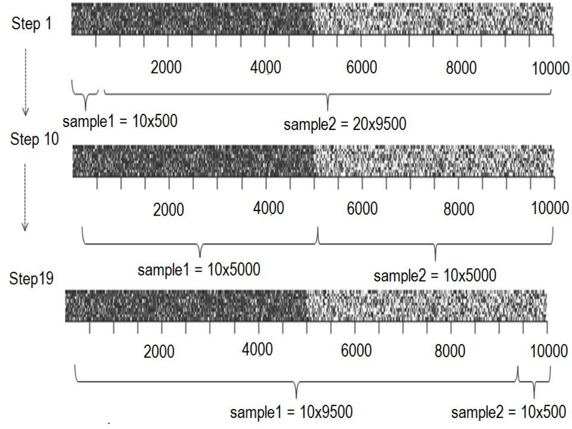
Each geodesic distance can be turned into a test statistic.

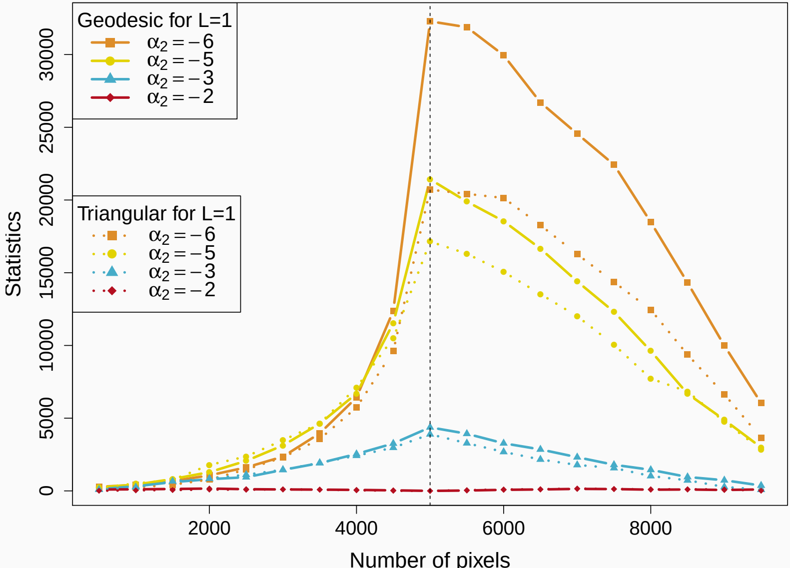
All these test statistics have known asymptotic distribution, so they are interpretable and comparable.

Moreover, they exhibit excellent finite-size behavior: they have good size and power even with very small samples.

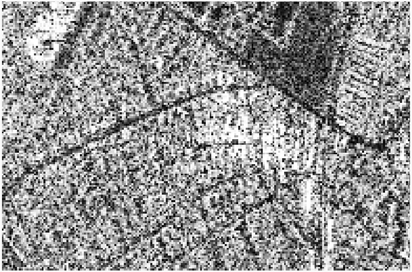
Edge Detection with GD under the $\mathcal{G}^0$ law

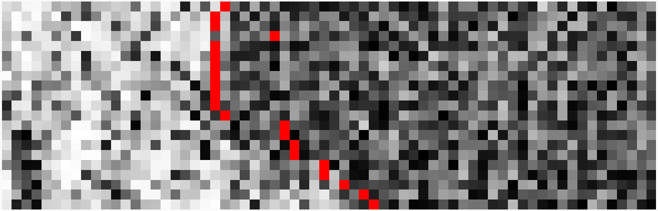
Naranjo-Torres et al. (2017) obtained expressions for the GD between $\mathcal{G}^0$ models and proposed a line search algorithm for edge detection.




Geodesic for L=1
α2 = −6
α2 = −5
α2 = −3
α2 = −2
Triangular for L=1
α2 = −6
α2 = −5
α2 = −3
α2 = −2
Statistics
0
5000
10000
15000
20000
25000
30000
2000
4000
6000
8000
Number of pixels

Edge Detection with GD





Outline

What is PolSAR?

The Complex Wishart Distribution

Analogously to the $\Gamma(\sigma^2, L)$ for intensity data, the Complex Wishart distribution is a widely accepted model of fully polarimetric data. It is characterized by the density

$$f_{\boldsymbol{Z}}(\boldsymbol{z}; \Sigma, L) = \frac{L^{pL}|\boldsymbol{z}|^{L-p}}{|\Sigma|^L \Gamma_p(L)} \exp\bigl[-L \operatorname{tr}\bigl(\Sigma^{-1}\boldsymbol{z}\bigr)\bigr],$$

where $\Gamma_p(L) = \pi^{p(p-1)/2} \prod_{i=0}^{p-1} \Gamma(L-i)$, $L \geq p$, and $\operatorname{tr}(\cdot)$ is the trace operator. We denote it by $\boldsymbol{Z} \sim \mathcal{W}(\Sigma, L)$.

This distribution satisfies $\mathrm{E}\{\boldsymbol{Z}\} = \Sigma$, which is a Hermitian positive definite matrix (Anfinsen et al., 2009). In practice, L is treated as a parameter and must be estimated, usually for the whole image.

Properties of the $\mathcal{W}$ distribution

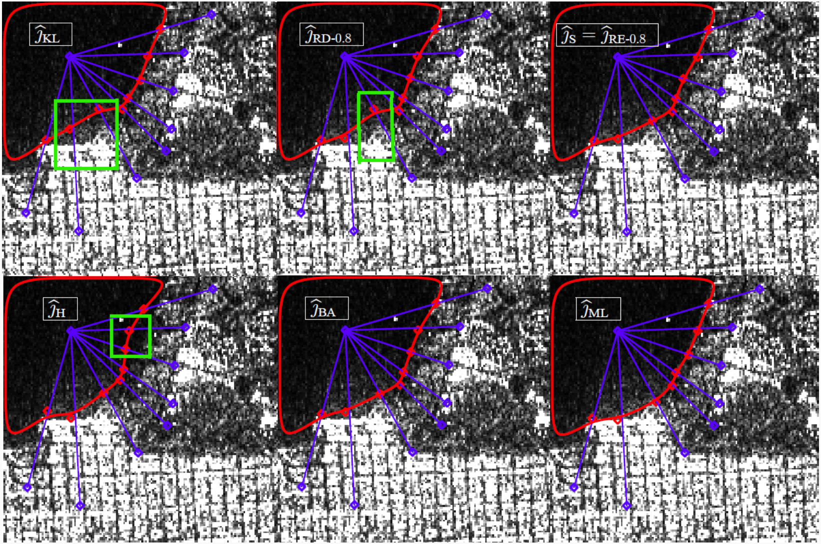
- It is a distribution for complex Hermitian matrices.
- The marginal distribution of each diagonal element is a $\Gamma(\sigma^2, L)$ law.
- The joint distribution of the diagonal elements was derived by Hagedorn et al. (2006).
- Other important marginal laws were obtained by Lee et al. (1994).

Tests based on divergences

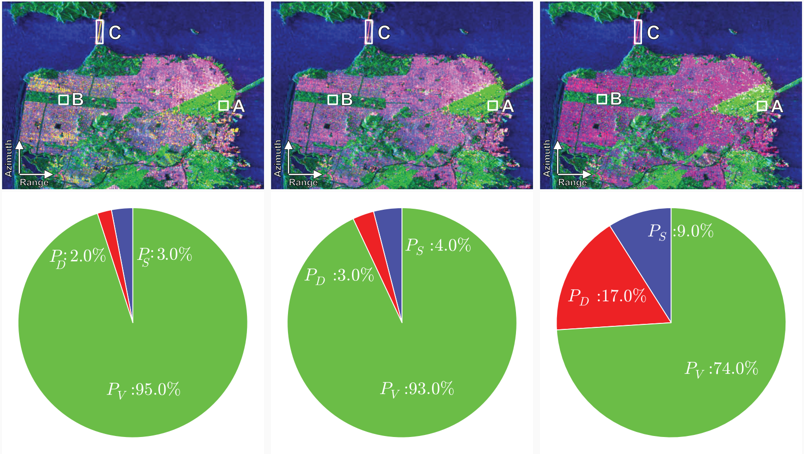
Frery et al. (2014) computed several of these tests under the $\mathcal{W}$ model:

- Relatively simple expressions
- They all rely only on the determinant and the inverse of Σ
- Good size and power even for very small (4) samples

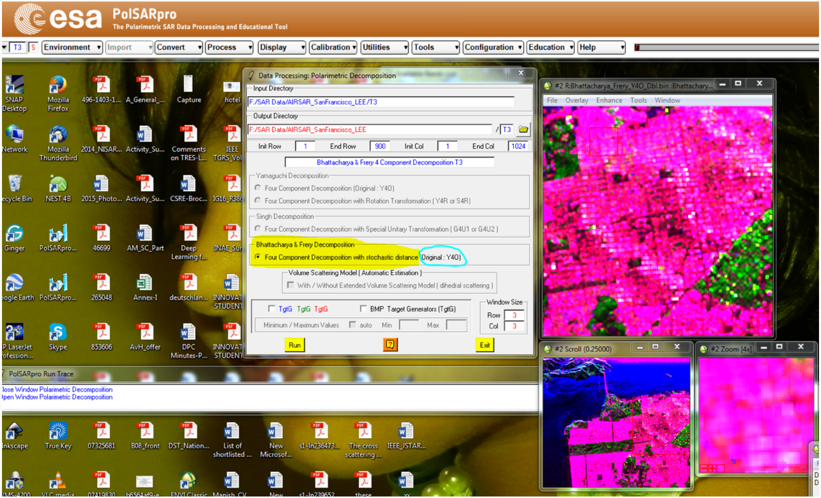
Edge Detection Stochastic Distances



Spectral Decomposition by Rotation

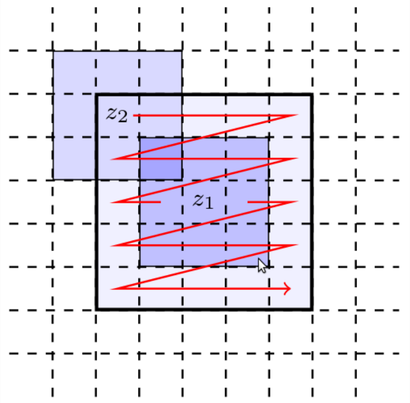


PolSARpro

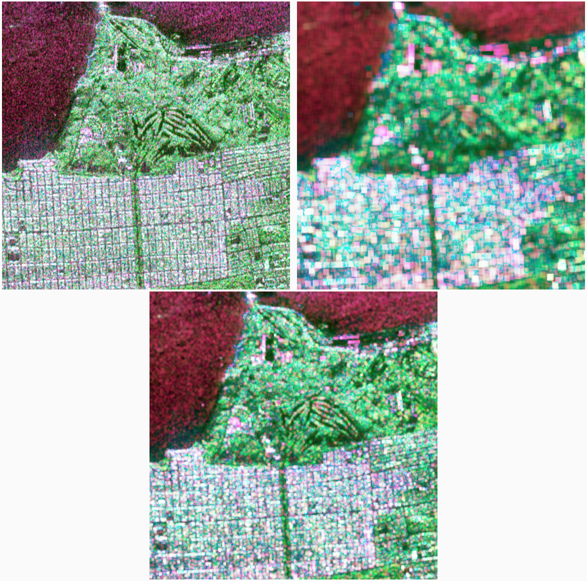


esa
PolSARpro
The Polarimetric SAR Data Processing and Educational Tool
Environment
Convert
Process
Display
Calibration
Utilities
Tools
Configuration
Education
Help
Data Processing: Polarimetric Decomposition
Input Directory
F:/SAR Data/AIRSAR_SanFrancisco_LEE/T3
Output Directory
F:/SAR Data/AIRSAR_SanFrancisco_LEE
Init Row
End Row
900
Init Col
End Col
1024
Bhattacharya & Frery 4 Component Decomposition T3
Yamaguchi Decomposition
Four Component Decomposition (Original : Y4O)
Four Component Decomposition with Rotation Transformation (Y4R or S4R)
Singh Decomposition
Four Component Decomposition with Special Unitary Transformation (G4U1 or G4U2)
Bhattacharya & Frery Decomposition
Four Component Decomposition with stochastic distance Original : Y4O)
Volume Scattering Model (Automatic Estimation)
With / Without Extended Volume Scattering Model (dihedral scattering)
BMP Target Generators (TgtG)
Window Size
Row
Col
Minimum / Maximum Values
auto
Min
Max
Run
Exit
PolSARpro Run Trace
#2 Scroll (0.25000)
#2 Zoom [4x]
File
Overlay
Enhance
Tools
Window

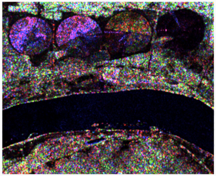
A large convolution mask is computed at each position, with weights inversely proportional to the dissimilarity between the point and the central value.



Nonlocal Means Speckle Reduction



From Segments to Classes



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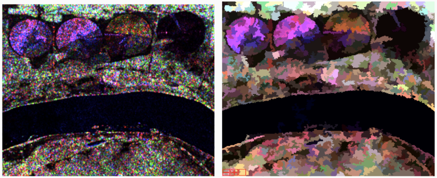

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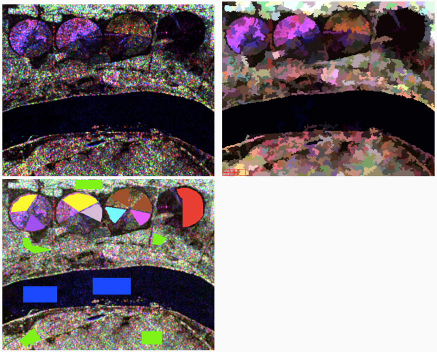
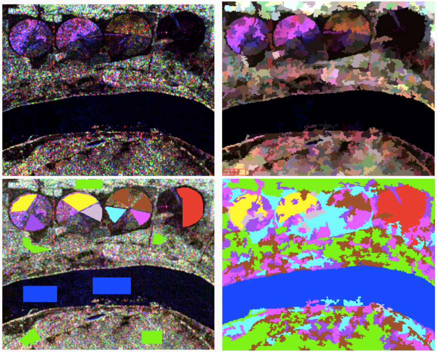


From Segments to Classes



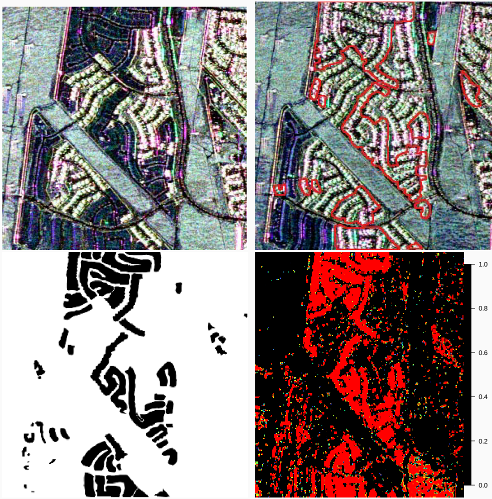
Difference of Entropies

It provides a powerful result for checking if at least one of the scenes under test has changed from the rest.

Being a test statistic, it is possible to compute the p-value of the no-change hypothesis at each position, rather than a binary mask.

This was applied to pairs of fully PolSAR images.

Change Detection



Outline

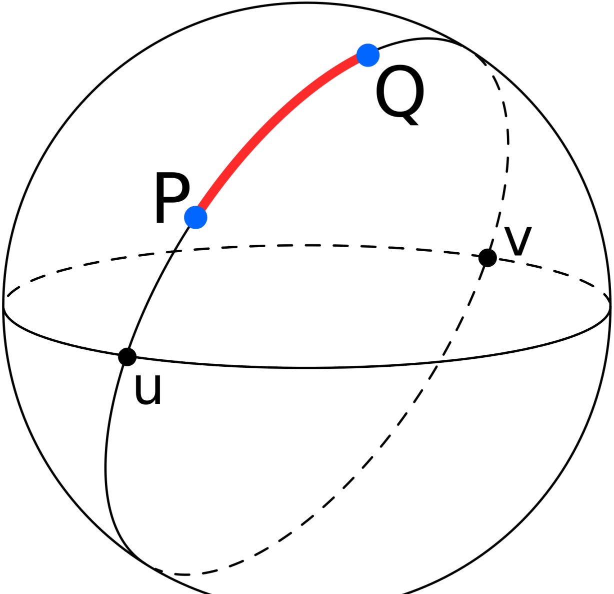
The frontiers: Kennaugh projection

Fully PolSAR data, and elementary backscatterers, can be projected onto the surface of a 16-dimensional sphere.

Then, distances become angles.

By measuring angles, we can classify, and decompose observations.

Once projected...



- Several interesting problems can be posed in the form of comparing samples and/or models.
- Information Theory and Information Geometry, along with Statistics, provide us with tools for solving such problems.
- Although the focus was on SAR/PolSAR, the basic ideas can be used whenever there is a model... even when there is no model!
- This is a fertile area for scientific research and technology development.

Research avenues I

We only presented the geodesic distance for p-dimensional parameters, which is seldom feasible.

The alternative we worked with consisted in dealing with a one-dimensional parameter after scaling the data.

There is an another approach which allows *mixing* one-dimensional test statistics and using other geodesic metrics. This approach is closely related to generalized entropies.

Research avenues II

- What happens with the convergence when we use non-ML estimators?
- What happens with the convergence when we use kernel estimators?
- Can we obtain other geodesic distances, stochastic distances, and difference of entropy, and their tests?
- Can we turn them into other useful techniques?
- What can we say about the statistics of distances in the Kennaugh space?

Allende, H., Frery, A. C., Galbiati, J. & Pizarro, L. (2006), 'M-estimators with asymmetric influence functions: the GA0 distribution case', *Journal of Statistical Computation and Simulation* **76**(11), 941–956.

Anfinsen, S. N., Doulgeris, A. P. & Eltoft, T. (2009), 'Estimation of the equivalent number of looks in polarimetric synthetic aperture radar imagery', *IEEE Transactions on Geoscience and Remote Sensing* **47**(11), 3795–3809.

Bhattacharya, A., Muhuri, A., De, S., Manickam, S. & Frery, A. C. (2015), 'Modifying the Yamaguchi four-component decomposition scattering powers using a stochastic distance', *IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing* **8**(7), 3497–3506.

Box, G. E., Hunter, J. S. & Hunter, W. G. (2005), *Statistics for Experimenters: design, discovery and innovation*, 2 ed., Wiley, Hoboken, NJ.

Bustos, O. H., Flesia, A. G. & Frery, A. C. (2001), 'Performance of spectral estimators in simulated synthetic aperture radar images', *Latin American Applied Research* **31**(2), 93–98. URL `http://www.laar.uns.edu.ar/indexes/i31_02.htm`.

Bustos, O. H., Lucini, M. M. & Frery, A. C. (2002), 'M-estimators of roughness and scale for GA0-modelled SAR imagery', *EURASIP Journal on Advances in Signal Processing* **2002**(1), 105–114.

Bustos, O. H., Ruiz, M., Ojeda, S., Vallejos, R. & Frery, A. C. (2009), 'Asymptotic behavior of RA-estimates in autoregressive 2D processes', *Journal of Statistical Planning and Inference* **139**, 3649–3664.

Cintra, R. J., Frery, A. C. & Nascimento, A. D. C. (2013), 'Parametric and nonparametric tests for speckled imagery', *Pattern Analysis and Applications* **16**(2), 141–161.

Cribari-Neto, F., Frery, A. C. & Silva, M. F. (2002), 'Improved estimation of clutter properties in speckled imagery', *Computational Statistics and Data Analysis* **40**(4), 801–824.

Ferral, A., Luccini, E., Solis, V., Frery, A. C., Aleksinko, A., Bernasconi, I. & Scavuzzo, C. M. (2018), 'In-situ and satellite monitoring of water quality of an eutrophic lake with an artificial air diffusion system', *IEEE Latin America Transactions* **16**(2), 627–633.

Freitas, C. C., Frery, A. C. & Correia, A. H. (2005), 'The polarimetric G distribution for SAR data analysis', *Environmetrics* **16**(1), 13–31.

Frery, A. C., Cintra, R. J. & Nascimento, A. D. C. (2013), 'Entropy-based statistical analysis of PolSAR data', *IEEE Transactions on Geoscience and Remote Sensing* **51**(6), 3733–3743.

Frery, A. C., Correia, A. H. & Freitas, C. C. (2007), 'Classifying multifrequency fully polarimetric imagery with multiple sources of statistical evidence and contextual information', *IEEE Transactions on Geoscience and Remote Sensing* **45**(10), 3098–3109.

Frery, A. C., Cribari-Neto, F. & Souza, M. O. (2004), 'Analysis of minute features in speckled imagery with maximum likelihood estimation', *EURASIP Journal on Advances in Signal Processing* **2004**(16), 2476–2491.

Frery, A. C., Jacobo-Berlles, J., Gambini, J. & Mejail, M. (2010), 'Polarimetric SAR image segmentation with B-Splines and a new statistical model', *Multidimensional Systems and Signal Processing* **21**, 319–342.

Frery, A. C., Müller, H.-J., Yanasse, C. C. F. & Sant'Anna, S. J. S. (1997), 'A model for extremely heterogeneous clutter', *IEEE Transactions on Geoscience and Remote Sensing* **35**(3), 648–659.

Frery, A. C., Nascimento, A. D. C. & Cintra, R. J. (2014), 'Analytic expressions for stochastic distances between relaxed complex Wishart distributions', *IEEE Transactions on Geoscience and Remote Sensing* **52**(2), 1213–1226.

Gambini, J., Cassetti, J., Lucini, M. M. & Frery, A. C. (2015), 'Parameter estimation in SAR imagery using stochastic distances and asymmetric kernels', *IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing* **8**(1), 365–375.

Gambini, J., Mejail, M., Jacobo-Berlles, J. & Frery, A. C. (2006), 'Feature extraction in speckled imagery using dynamic B-spline deformable contours under the G0 model', *International Journal of Remote Sensing* **27**(22), 5037–5059.

Gambini, J., Mejail, M., Jacobo-Berlles, J. & Frery, A. C. (2008), 'Accuracy of edge detection methods with local information in speckled imagery', *Statistics and Computing* **18**(1), 15–26.

Girón, E., Frery, A. C. & Cribari-Neto, F. (2012), 'Nonparametric edge detection in speckled imagery', *Mathematics and Computers in Simulation* **82**, 2182–2198. URL http://www.sciencedirect.com/science/article/pii/S037847541200136X.

Gomez, L., Alvarez, L., Mazorra, L. & Frery, A. C. (2015), 'Classification of complex Wishart matrices with a diffusion-reaction system guided by stochastic distances', *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences* **373**(2056), 20150118. URL http://dx.doi.org/10.1098/rsta.2015.0118.

Gomez, L., Alvarez, L., Mazorra, L. & Frery, A. C. (2017*a*), 'Fully PolSAR image classification using machine learning techniques and reaction-diffusion systems', *Neurocomputing* **255**, 52–60.

Gomez, L., Ospina, R. & Frery, A. C. (2017*b*), 'Unassisted quantitative evaluation of despeckling filters', *Remote Sensing* **9**(9), 389. URL https://github.com/Raydonal/UNASSISTED.

Gomez, L., Ospina, R. & Frery, A. C. (2019), 'Statistical properties of an unassisted image quality index for SAR', *Remote Sensing* **11**(4), 1–16.

Hagedorn, M., Smith, P. J., Bones, P. J., Millane, R. P. & Pairman, D. (2006), 'A trivariate chi-squared distribution derived from the complex Wishart distribution', *Journal of Multivariate Analysis* **97**, 655–674.

Lee, J.-S. & Pottier, E. (2009), *Polarimetric Radar Imaging: From Basics to Applications*, CRC, Boca Raton.

Lee, J. S., Hoppel, K. W., Mango, S. A. & Miller, A. R. (1994), 'Intensity and phase statistics of multilook polarimetric and interferometric SAR imagery', *IEEE Transactions on Geoscience and Remote Sensing* **32**(5), 1017–1028.

Lucini, M. M., Ruiz, V. F., Frery, A. C. & Bustos, O. H. (2003), Robust classification of SAR imagery, *in* 'IEEE International Conference on Acoustics, Speech and Signal Processing', IEEE, Hong Kong, pp. 557–560.

Mejail, M. E., Jacobo-Berlles, J., Frery, A. C. & Bustos, O. H. (2000), 'Parametric roughness estimation in amplitude SAR images under the multiplicative model', *Revista de Teledetección* **13**, 37–49.

Mejail, M. E., Jacobo-Berlles, J., Frery, A. C. & Bustos, O. H. (2003), 'Classification of SAR images using a general and tractable multiplicative model', *International Journal of Remote Sensing* **24**(18), 3565–3582.

Moschetti, E., Palacio, M. G., Picco, M., Bustos, O. H. & Frery, A. C. (2006), 'On the use of Lee's protocol for speckle-reducing techniques', *Latin American Applied Research* **36**(2), 115–121.

Naranjo-Torres, J., Gambini, J. & Frery, A. C. (2017), 'The geodesic distance between GI0 models and its application to region discrimination', *IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing* **10**(3), 987–997.

Nascimento, A. D. C., Cintra, R. J. & Frery, A. C. (2010), 'Hypothesis testing in speckled data with stochastic distances', *IEEE Transactions on Geoscience and Remote Sensing* **48**(1), 373–385.

Nascimento, A. D. C., Frery, A. C. & Cintra, R. J. (2014*a*), 'Bias correction and modified profile likelihood under the Wishart complex distribution', *IEEE Transactions on Geoscience and Remote Sensing* **52**(8), 4932–4941.

Nascimento, A. D. C., Frery, A. C. & Cintra, R. J. (2019), 'Detecting changes in fully polarimetric SAR imagery with statistical information theory', *IEEE Transactions on Geoscience and Remote Sensing* **57**(3), 1380–1392.

Nascimento, A. D. C., Horta, M. M., Frery, A. C. & Cintra, R. J. (2014*b*), 'Comparing edge detection methods based on stochastic entropies and distances for PolSAR imagery', *IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing* **7**(2), 648–663.

Negri, R. G., Frery, A. C., Silva, W. B., Mendes, T. S. G. & Dutra, L. V. (2019), 'Region-based classification of PolSAR data using radial basis kernel functions with stochastic distances', *International Journal of Digital Earth* **12**(6), 699–719. URL `https://doi.org/10.1080/17538947.2018.1474958`.

Palacio, M. G., Ferrero, S. B. & Frery, A. C. (2019), 'Revisiting the effect of spatial resolution on information content based on classification results', *International Journal of Remote Sensing* **40**(12), 4489–4505. URL `https://doi.org/10.1080/01431161.2019.1569278`.

Penna, P. A. A. & Mascarenhas, N. D. A. (2019), 'SAR speckle nonlocal filtering with statistical modeling of Haar wavelet coefficients and stochastic distances', *IEEE Transactions on Geoscience and Remote Sensing* pp. 1–15.

Ratha, D., Bhattacharya, A. & Frery, A. C. (2018), 'Unsupervised classification of PolSAR data using a scattering similarity measure derived from a geodesic distance', *IEEE Geoscience and Remote Sensing Letters* **15**(1), 151–155.

Ratha, D., Gamba, P., Bhattacharya, A. & Frery, A. C. (2019*a*), 'Novel techniques for built-up area extraction from polarimetric SAR images', *IEEE Geoscience and Remote Sensing Letters* p. in press.

Ratha, D., Mandal, D., Kumar, V., McNairn, H., Bhattacharya, A. & Frery, A. C. (2019*b*), 'A generalized volume scattering model based vegetation index from polarimetric SAR data', *IEEE Geoscience and Remote Sensing Letters* p. in press.

Santana-Cedrés, D., Gomez, L., Alvarez, L. & Frery, A. C. (2019), 'Despeckling PolSAR images with a structure tensor filter', *IEEE Geoscience and Remote Sensing Letters* p. in press.

Silva, M., Cribari-Neto, F. & Frery, A. C. (2008), 'Improved likelihood inference for the roughness parameter of the GA0 distribution', *Environmetrics* **19**(4), 347–368. URL http://www3.interscience.wiley.com/cgi-bin/abstract/114801264/ABSTRACT.

Silva, W. B., Freitas, C. C., Sant'Anna, S. J. S. & Frery, A. C. (2013), 'Classification of segments in PolSAR imagery by minimum stochastic distances between Wishart distributions', *IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing* **6**(3), 1263–1273.

Torres, L., Sant'Anna, S. J. S., Freitas, C. C. & Frery, A. C. (2014), 'Speckle reduction in polarimetric SAR imagery with stochastic distances and nonlocal means', *Pattern Recognition* **47**, 141–157.

Vasconcellos, K. L. P., Frery, A. C. & Silva, L. B. (2005), 'Improving estimation in speckled imagery', *Computational Statistics* **20**(3), 503–519.

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