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Una dualidad topológica para álgebras de boole bitopológicas

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Las Álgebras de Boole Topológicas, llamadas también "Álgebras Interiores" o "Álgebras de Clausura", fueron estudiadas por Tarski [5] y Naturman [2]. Los Espacios Bitopológicos, a su vez, fueron estudiados por [1]. En [3], se presentan las Álgebras de Boole Bitopológicas, que relacionan y generalizan naturalmente las estructuras anteriormente mencionadas. Entre otras aplicaciones, permiten dar un enfoque topológico a la completación de Dedekind-MacNeille de Álgebras de Heyting [4]. Definiremos explícitamente Funtores que establecen una equivalencia natural entre las categorías de Álgebras de Boole Bitopológicas y las de ciertos Espacios Topológicos.

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Demiquantifiers on MV-algebras

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In this paper we introduce the notion of *demiquantifiers* on MV-algebras. Given an MV-algebra A, let $A^- = \{x \in A : x \leq \neg x\}$ and let $A^+ = \{x \in A : x \geq \neg x\}$. Demiquantifiers determine a new type of quantifiers on MV-algebras. These operators behave like an existential quantifier when restricted to the set A^- and like an universal quantifier when restricted to A^+ where A is an arbitrary MV-algebra. A particular and interesting case is constructed in the functional MV-algebra $[0,1]^X$ determined by all functions defined on a non-empty set X with values in the real unit interval $[0,1]$. In this case we define a demiquantifier $\exists_{\frac{1}{2}}$ having the following semantic interpretation: given a propositional function $f : X \to [0,1]$, then $\exists_{\frac{1}{2}} f = \frac{1}{2}$ if and only if $f^{-1}(\frac{1}{2} - \varepsilon, \frac{1}{2} + \varepsilon) \neq \emptyset$ for every $\varepsilon > 0$. In particular, when the image of f is a finite subset of $[0,1]$, then $\exists_{\frac{1}{2}} f = \frac{1}{2}$ if and only if $f(x_0) = \frac{1}{2}$ for some $x_0 \in X$. Taking into account that the classical existential quantifier $\exists$ satisfies $\exists f = 1$ if and only if $f^{-1}(1-\varepsilon, 1] \neq \emptyset$ for every $\varepsilon > 0$ and the universal quantifier $\forall$ satisfies $\forall f = 0$ if and only if $f^{-1}[0,\varepsilon) \neq \emptyset$ for every $\varepsilon > 0$, then the demiquantifier $\exists_{\frac{1}{2}}$ can be considered as a non-classical quantifier satisfying the corresponding property for the *neighborhoods* of the constant $\frac{1}{2}$. We prove that these quantifiers determine a variety and are interdefinable with the usual existential quantifiers on MV-algebras given by DiNola and Grigolia ([1]) provided the corresponding MV-algebras have a fixed point. Moreover, demiquantifiers generalize the notion of existenctial middle quantifier considered in [2] for the class of three-valued Łukasiewicz algebras.

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Semantics of triples for the first-order paraconsistent logic QCiore

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In this talk the logic **QCiore** is introduced as a first-order version of the 3-valued paraconsistent proposicional logic **LFI2**, introduced in [2] and additionally studied in [1] under the name of **Ciore**. As semantical counterpart for **QCiore** we consider the so-called **LFI2**-structures, which are defined as $\langle \mathfrak{A}, \|\cdot\|_{\mathbf{LFI2}} \rangle$, such that $\mathfrak{A}$ is a partial structure as introduced in [3], and $\|\cdot\|_{\mathbf{LFI2}}$ is a mapping which assigns to each formula of the language of **QCiore** $\mathscr{M}$-fuzzy set (a concept taken from [4]) over the set of all the variable assignments in the domain of $\mathfrak{A}$, where $\mathscr{M}$ is the logical matrix of **Ciore**.

Soundness and Completeness theorems for **QCiore** with respect to **LFI2**-structures are obtained, and a first study of **LFI2**-structures from the point of view of Model theory is developed.

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A model-theoretic study of the class of F-structures for mbC

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As it is well-known, many paraconsistent logic in the class of the *Logics of Formal Inconsistency* (in short LFIs, see [4]) do not allow us an algebrization by means of Blok-Pigozzi's method. However, some of them can be semantically characterized by non-deterministic structures such as Nmatrices and possible-translations semantics. A semantic of Nmatrices based on an special kind of multialgebra called *swap structures* was proposed in [2, 3] and an algebraic study of them was developed in [7].

The decidability of da Costa's calculi C_n (an special class of LFIs) was proved for the first time by M. Fidel in [8] by means of a novel algebraic-relational class of structures called *C_n-structures*. A C_n-structure is a triple $\langle \mathscr{A}, \{N_a\}_{a\in A}, \{N_a^{(n)}\}_{a\in A}\rangle$ such that $\mathscr{A}$ is a Boolean algebra with domain A and each N_a and $N_a^{(n)}$ is a non-empty subset of A. The intuitive meaning of $b \in N_a$ and $c \in N_a^{(n)}$ is that b and c are possible values for the paraconsistent negation $\neg a$ of a and for the well-behavior a° of a, respectively. This kind of structure was baptized by S. Odintsov as *Fidel structures* or **F**-structures. In [2, 3], a semantics of **F**-structures was found for **mbC** and several of its extensions.

In this talk, an initial study of the class of **F**-structures for **mbC** (for short, mbC-structures) from the point of view of Model Theory is proposed. From this perspective, the **F**-structures are seen as first-order structures over the signature of Boolean algebras expanded with two binary predicate symbols N and O for the paraconsistent negation and the consistency connective, respectively. As a consequence of this, notions and tools from Model Theory and Category Theory can be applied. This point of view allows us to consider notions such as substructures, union of chains, direct products, direct limits, congruences, quotient structures, ultraproducts, etc. In this sense, an result due to X. Caicedo ([1]) will be used. In that paper, conditions under which it is possible to have a Birkhoff-like theorem about subdirectly decomposition for first-order structures of classical model Theory are provided. Using this, a representation theorem for mbC-structures as subdirect poducts of subdirectly irreducible (s.i.) mbC-structures is obtained by adapting Caicedo's results. In particular, given an mbC-structure over the two-atoms Boolean algebra we can determine whether it is s.i. or not. Besides, every mbC-structure over the two-element Boolean algebra is proved to be subdirectly irreducible.

Finally, another decomposition theorem is obtained by using the notions of *weak substructure* and *weak isomorphism* by adapting results due to Fidel for C_n-structures. In this case, the mbC-structures over the two-element Boolean algebra play the role of s.i. structures.

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Definibilidad por fórmulas abiertas en estructuras relacionales

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En esta charla hablaremos sobre el siguiente problema computacional:

OpenDef:: Toma como entrada una estructura finita **A** y una relación T sobre el universo de **A**, y determina si hay una fórmula abierta tal que su extensión coincida con T.

Mostraremos que su complejidad es coNP-complete y luego presentarmos un algoritmo que toma ventaja de una caracterización semántica de la definibilidad abierta. Finalizaremos mostrando una implementación del algoritmo y su uso con algunos ejemplos.

Epistemic BL-algebras
An Algebraic Semantics for BL-Possibilistic Logic

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Possibilistic logic [4, 2] is a well-known uncertainty logic to reasoning with graded beliefs on classical propositions by means of necessity and possibility measures. When trying to extend the possibilistic belief model beyond the classical framework of Boolean propositions to many-valued propositions, one has to come up with appropriate extensions of the notion of necessity and possibility measures for them. In the particular context of BL-fuzzy logic [3], a natural generalization that we will consider is the following: given a BL-algebra $\mathscr{A}$, a possibility distribution $\pi : \Omega \mapsto \mathscr{A}$ on the set Ω of $\mathscr{A}$-propositional interpretations we define the following generalized possibility and necessity measures over $\mathscr{A}$-propositions:

$$\Pi(\varphi) = \sup_{w\in\Omega}\{\pi(w) * w(\varphi)\},\ N(\varphi) = \text{ínf}_{w\in\Omega}\{\pi(w) \to w(\varphi)\}$$

where $*, \to$ are the product and the implication in $\mathscr{A}$. In this setting, W is a non-empty set of worlds, $\pi : W \mapsto \mathscr{A}$ is a possibility distribution on W, and $e : Var \times W \mapsto \mathscr{A}$ is a $\mathscr{A}$-propositional evaluation in each world. The set of valid formulas in the class $\Pi\mathscr{A}$ of *possibilistic Kripke models*, is denoted by $Val(\Pi\mathscr{A})$. Finding an axiomatic characterization of $Val(\Pi\mathscr{A})$ is an open problem proposed by Hájek in Chapter 8 of [3].

With the intend to solve this open problem by introducing the variety of *Epistemic BL-algebras* as BL-algebras endowed with two operators $\forall$ and $\exists$, considering that the models resulting from the Kripke semantics are EBL-algebras. In this talk we are going to introduce EBL-algebras, we will show some examples and properties of them. We will also compare them with monadic BL-algebras as defined in [3].

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Relation Between BL-Possilistic Logic and Epistemic BL-Algebras

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Possibilistic logic [2] is a well-known uncertainty logic to reasoning with graded beliefs on classical propositions by means of necessity and possiblity measures. From a logical point of view, possibilistic logic can be seen as a graded extension of the well-know modal logic of belief KD45. When we go beyond the classical framework of Boolean algebras of events to BL-algebras frameworks, one has to come up with appropriate extensions of the notion of necessity and possibility measures for BL-valued events. In current work, we consider the general problem of giving an axiomatization of BL-Possibilistic Logic. The particular case when BL is a Gödel algebra was solved in [1]. However in the general setting of BL-algebras that it is an open problem proposed by Hájek in Chapter 8 of [3]. In our presentation, this problem is addressed and solved for the particular case of finite BL-algebras. For that, we introduce a connection between BL-possibilistic models and Epistemic BL-algebras which is showed to be an algebriac semantics for BL-possibilistic logic.

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On Kalman's functor for bounded hemiimplicative semilattices and hemiimplicative lattices

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A hemiimplicative semilattice is an algebra $(H,\wedge,\rightarrow,1)$ of type $(2,2,0)$ such that $(H,\wedge)$ is a meet semilattice, 1 is the greatest element with respect to the order, $a\rightarrow a=1$ for every $a\in H$ and for every a, b, $c\in H$, if $a\leq b\rightarrow c$ then $a\wedge b\leq c$. A bounded hemiimplicative semilattice is an algebra $(H,\wedge,\rightarrow,0,1)$ of type $(2,2,0,0)$ such that $(H,\wedge,\rightarrow,1)$ is a hemiimplicative semilattice and 0 is the first element with respect to the order. A hemiimplicative lattice is an algebra $(H,\wedge,\vee,\rightarrow,0,1)$ of type $(2,2,2,0,0)$ such that $(H,\wedge,\vee,0,1)$ is a bounded distributive lattice and $(H,\wedge,\rightarrow,1)$ is a hemiimplicative semilattice.

In this work we study an equivalence for the categories of bounded hemiimplicative semilattices and hemiimplicative lattices, respectively, which is motivated by an old construction due J. Kalman that relates bounded distributive lattices and Kleene algebras.

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A canonical representation for free left-dioids

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Dioids (or *cubical monoids*) were introduced by Marco Grandis [1] in an attempt to decategorify the notion of cubical monad. These algebras can be relaxed in such a way that only left or right absorptions are assumed. Concretely, a *left-dioid* is an algebra $\mathbf{D}=\langle D,\oplus,\otimes,0,1\rangle$ that satisfies the conditions:

(D1) $\langle D,\oplus,0\rangle$ is a monoid
(D2) $\langle D,\otimes,1\rangle$ is a monoid
(D3) $0\otimes x\approx 0$
(D4) $1\oplus x\approx 1$

Motivated by categorical interpretations of functional programs, we found left-dioids related to computational effects in the presence of exceptions. In this work we introduce a canonical representation for the free left-dioid over a set, obtaining an implementation which can be presented in a language with a simple type system.

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