A Categorical equivalence for Stonean distributive residuated lattices.

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based on a joint work with R. Cignoli and M. Marcos

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such that

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- $\mathbf{L}(\mathbf{A}) = \langle A; \vee, \wedge, \top \rangle$ is a lattice with greatest element $\top$,
- the following residuation condition holds:

$$x * y \leq z \text{ iff } x \leq y \rightarrow z \tag{1}$$

Bounded residuated lattices

A bounded residuated lattice is an algebra

$$\mathbf{A} = \langle A, *, \rightarrow, \vee, \wedge, \top, \bot \rangle$$

such that $\langle A, *, \rightarrow, \vee, \wedge, \top \rangle$ is a residuated lattice, and $\bot$ is the smallest element of the lattice $\mathbf{L}(\mathbf{A})$.

Famous bounded residuated lattices

- Boolean algebras
- Heyting algebras
- MV-algebras
- BL-algebras
- MTL-algebras
- NM-algebras

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- $x \wedge \neg x = \bot$
- $\neg x \vee \neg\neg x = \top$

Stone lattices

A distributive Stonean residuated lattice is a residuated lattice $\mathbf{A}$ whose lattice reduct $\mathbf{L}(\mathbf{A})$ is a Stone lattice.

Stonean residuated lattices

Stonean residuated lattices are bounded residuated lattices satisfying

$$\neg x \vee \neg\neg x = \top.$$

Boolean skeleton

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B(**A**) is a subalgebra of **A** which is a Boolean algebra.

Importance of the Boolean skeleton in Stonean residuated lattices

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(i) $\mathbf{A}$ *is Stonean,*

(ii) $B(A) \supseteq \neg(A) := \{\neg x : x \in A\}$.

Theorem

$\mathbf{A}$ *is a Stonean residuated lattice if and only if the application:*

$$h: \ \mathbf{A} \to \mathbf{B}(\mathbf{A})$$

$$x \mapsto \neg\neg x$$

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is a retraction, i.e., it satisfies that

1. *h is an onto homomorphism*
2. *and $h(h(x)) = h(x)$ for each $x \in A$.*

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Distributive Stonean residuated lattices (*DSRL*) are:

1. The greatest subvariety of bounded distributive residuated lattices such that $\neg\neg$ is a retraction onto the Boolean skeleton
2. Bounded distributive residuated lattices that satisfy $\neg x \vee \neg\neg x = \top$.
3. Bounded distributive residuated lattices such that their lattice reducts are Stone algebras (lattices).

Famous distributive Stonean residuated lattices

- Boolean algebras
- Pseudocomplemented BL-algebras
- Product algebras
- Gödel algebras
- Pseudocomplemented MTL-algebras
- Stonean Heyting algebras

Dense elements

Let **A** be in *DSRL*. Since $\neg\neg : A \to B(A)$ is a retraction, the kernel,

$$D(A) = \{x \in A : \neg\neg x = \top\}$$

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is a filter of $\mathbf{A}$.

We will consider

$$\mathbf{D}(\mathbf{A}) = (D(A), *, \to, \vee, \wedge, \top)$$

as an integral distributive residuated lattice.

- How can we use the information of $\mathbf{B}(\mathbf{A})$ and of $\mathbf{D}(\mathbf{A})$ to characterize the algebra $\mathbf{A}$?

Building algebras in *DSRL*

Let **D** be an integral distributive residuated lattice and an element $o \notin D$.

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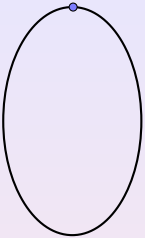
Let **D** be an integral distributive residuated lattice and an element $o \notin D$.

Adjoining the element o as bottom element, then

$$\mathbf{S}(\mathbf{D}) = (\{o\} \cup D, *, \to, \vee, \wedge, \top, o)$$

is in *DSRL*



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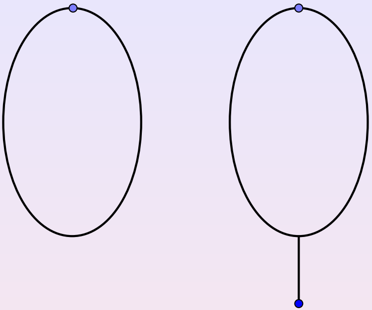

$\mathbf{D}$ $\qquad\qquad$ $S(\mathbf{D})$

Theorem

Given a distributive residuated lattice $\mathbf{D}$*, then*

1. $\mathbf{S}(\mathbf{D})$ *is a Stonean residuated lattice,*
2. $B(\mathbf{S}(\mathbf{D})) = \{\bot, \top\}$.
3. $D(\mathbf{S}(\mathbf{D})) = \mathbf{D}$
4. *Each homomorphism* $h : \mathbf{D}_1 \to \mathbf{D}_2$ *can be extended to a homomorphism* $\mathbf{S}(h) : \mathbf{S}(\mathbf{D}_1) \to \mathbf{S}(\mathbf{D}_2)$ *by the prescription*

$$\mathbf{S}(h)(x) = \begin{cases} h(x) & \text{if } x \in D_1, \\ o_{\mathbf{S}(D_2)} & \text{if } x = o_{\mathbf{S}(D_1)}. \end{cases}$$

The functor

$$\mathbf{S} : \mathit{RL} \to \mathit{diDSRL}$$

given by

$$\mathbf{D} \mapsto \mathbf{S}(\mathbf{D}) \quad \text{and} \quad h \mapsto \mathbf{S}(h)$$

establish a categorical equivalence.

Isomorphic Boolean skeletons and Isomorphic dense elements

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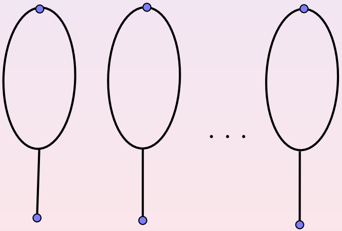

$\mathbf{B}(\mathbf{A}_1) \cong \prod_{\mathbb{N}}\{\perp, \top\}$ and $\mathbf{D}(\mathbf{A}_1) \cong \prod_{\mathbb{N}} \mathbf{D}$

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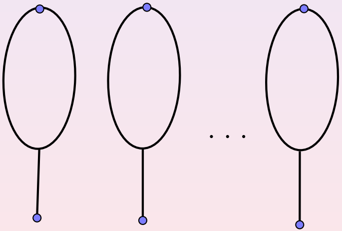

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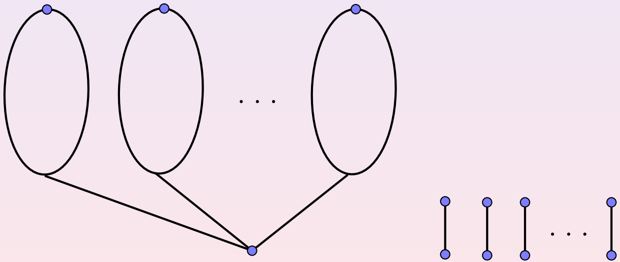
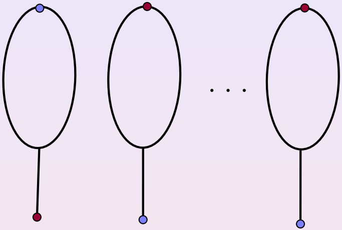


$\mathbf{B}(\mathbf{A}_2) \cong \prod_{\mathbb{N}} \{\bot, \top\}$ and $\mathbf{D}(\mathbf{A}_2) \cong \prod_{\mathbb{N}} \mathbf{D}$

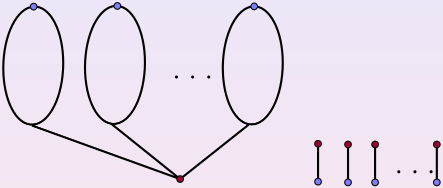
$$\mathbf{A}_1 \ncong \mathbf{A}_2.$$

How can we distinguish these two algebras?

Let $b \in B(\mathbf{A}_1)$. Then $[\neg b) \cap \mathbf{D}(\mathbf{A}_1) \cong \prod_{\mathbb{N}} \mathbf{D}$.



Let $b \in B(\mathbf{A}_2)$. Then $[\neg b) \cap \mathbf{D}(\mathbf{A}_2) \cong \{\top\}$



$$\phi_{\mathbf{A}} : \mathbf{B}(\mathbf{A}) \to Fi(\mathbf{D}(\mathbf{A}))$$

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$$\phi_{\mathbf{A}}(b) = \{x \in D(A) : x \geq \neg b\}.$$

Representation of elements

Let **A** be in *DSRL*. For each $x \in A$

$$x = \neg\neg x * (\neg\neg x \to x)$$

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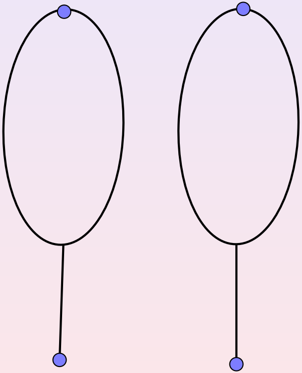
Let **A** be in *DSRL*. For each $x \in A$

$$x = \underbrace{\neg\neg x}_{B(A)} * \underbrace{(\neg\neg x \to x)}_{D(A)}$$

Consider $\mathbf{D}$ a residuated lattice and $\mathbf{A} \cong \mathbf{S}(\mathbf{D}) \times \mathbf{S}(\mathbf{D})$.



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For $d \in \mathbf{D}$ take

$$x = (\perp, d) \in \mathbf{A}.$$

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But for any $d' \in \mathbf{D}$ we also have

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$(\top, d)$ is the unique dense satisfying the representation that belongs to

$$\phi(\bot, \top) = \{(d_1, d_2) \in \mathbf{D}^2 : (d_1, d_2) \geq \neg(\bot, \top)\} =$$

$$\{(d_1, d_2) \in \mathbf{D}^2 : (d_1, d_2) \geq (\top, \bot)\}$$

Objects: Triples $(\mathbf{B}, \mathbf{D}, \phi)$ such that:

- $\mathbf{B}$ is a Boolean algebra,
- $\mathbf{D}$ is a distributive residuated lattice and
- ϕ is bounded lattice-homomorphism,

$$\phi : \mathbf{B} \to F_i(\mathbf{D}).$$

Morphisms: Given triples $(\mathbf{B}_i, \mathbf{D}_i, \phi_i)$, $i = 1, 2$, a morphism is a pair

$$(h, k) : (\mathbf{B}_1, \mathbf{D}_1, \phi_1) \to (\mathbf{B}_2, \mathbf{D}_2, \phi_2)$$

is a pair such that:

The category $\mathcal{T}$

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is a pair such that:

1. $h : \mathbf{B}_1 \to \mathbf{B}_2$ is a Boolean algebra homomorphism,
2. $k : \mathbf{D}_1 \to \mathbf{D}_2$ is a residuated lattice homomorphism, and
3. For all $a \in B_1$, $k(\phi_1(a)) \subseteq \phi_2(h(a))$.

The functor $\mathbf{T}$

$$\mathbf{T} : DSRL \to \mathcal{T}$$

$$\mathbf{A} \quad \mapsto \quad (\mathbf{B}(\mathbf{A}), \mathbf{D}(\mathbf{A}), \phi_A)$$

$$f : \mathbf{A}_1 \to \mathbf{A}_2 \quad \mapsto \quad \begin{array}{l} h : \mathbf{B}(\mathbf{A}_1) \to \mathbf{B}(\mathbf{A}_2) \\ \\ k : \mathbf{D}(\mathbf{A}_1) \to \mathbf{D}(\mathbf{A}_2) \end{array}$$

We need to prove that **T** is:

1. faithful
2. full
3. essentially surjective

Some ideas

That **T** is faithful follows immediate from the representation of each element x in $A \in DSRL$ by

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Let $\mathbf{T}(f_1) = (h_1, k_1)$ be equal to $\mathbf{T}(f_2) = (h_2, k_2)$.Then

$$f_1(x) = f_1(\neg\neg x * (\neg\neg x \to x)) = f_1(\neg\neg x) * f_1(\neg\neg x \to x) =$$

$$h_1(\neg\neg x) * k_1(\neg\neg x \to x) = h_2(\neg\neg x) * k_2(\neg\neg x \to x) =$$

$$= f_2(\neg\neg x) * f_2(\neg\neg x \to x) = f_2(\neg\neg x * (\neg\neg x \to x)) = f_2(x).$$

Some ideas

To see that **T** is full, given

$$(h, k) : \mathbf{T}(\mathbf{A}_1) \to \mathbf{T}(\mathbf{A}_2)$$

we have

$$h : \mathbf{B}(\mathbf{A}_1) \to \mathbf{B}(\mathbf{A_2}) \quad \text{and} \quad k : \mathbf{D}(\mathbf{A}_1) \to \mathbf{D}(\mathbf{A}_2).$$

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We define $f : \mathbf{A}_1 \to \mathbf{A}_2$ by

$$f(x) = h(\neg\neg x) * k(\neg\neg x \to x)$$

and we prove that f is a morphism such that $\mathbf{T}(f) = (h, k)$.

Some ideas

To see that $\mathbf{T}$ is essentially surjective (dense), for each triple $(\mathbf{B}, \mathbf{D}, \phi)$ we need to find an algebra $\mathbf{A}$ such that

$$\mathbf{T}(\mathbf{A}) = (\mathbf{B}(\mathbf{A}), \mathbf{D}(\mathbf{A}), \phi_{\mathbf{A}}) \cong (\mathbf{B}, \mathbf{D}, \phi)$$

Let $(\mathbf{B}, \mathbf{D}, \phi)$.

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$$\alpha : C(X) \to F_i(\mathbf{D})$$

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Moreover, we have

$$\rho_{a,b} : \mathbf{D}/\alpha(b) \to \mathbf{D}/\alpha(a)$$

the natural projection

Thus the system

$$R = \langle \{\mathbf{D}/\alpha(a)\}_{a \in C(X)}, \{\rho_{ab}\}_{a \subseteq b} \rangle$$

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Because of the categorical equivalence, the system

$$S = \langle \{\mathbf{S}(\mathbf{D}/\alpha(a))\}_{a \in C(X)}, \{\mathbf{S}(\rho_{ab})\}_{a \subseteq b} \rangle$$

is a presheaf of directly indecomposable Stonean residuated lattices.

For each $x \in X$,

$$F_x = \bigvee_{a \in C(x)} \alpha(a).$$

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Then $\mathbf{D}/F_x$ is the inductive limit of the system R and $\mathbf{S}(\mathbf{D})/F_x$ is the inductive limit of the system S.

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Since $\bigcap_{x \in X} F_x = \{\top\}$ the algebra $\mathbf{D}$ is a subdirect product of the family

$$\{\mathbf{D}/F_x\}_{x \in X}.$$

Now let

$$\mathcal{S} = \bigcup_{x \in X} (\{x\} \times \mathbf{S}(\mathbf{D}/F_x)),$$

and for each $x \in X$, $d \in D$ and $a \in C(X)$ let

$$\hat{d}(x) = \langle x, d/F_x \rangle \quad \hat{a}(x) = \begin{cases} \langle x, o_{\mathbf{S}(\mathbf{D}/F_x)} \rangle & \text{if } x \in a, \\ \langle x, \top \rangle & \text{if } x \in X \setminus a. \end{cases}$$

Now let

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Equipping $\mathcal{S}$ with the topology having as basis the sets

$$\{\hat{d}(x) : x \in a\} \text{ and } \{\hat{a}(x) : x \in a\}$$

and defining $\pi : \mathcal{S} \to X$ as the projection in the first coordinate.

The algebra of global sections

Theorem

$\langle \mathcal{S}, \pi, X \rangle$ is the sheaf of directly indecomposable Stonean residuated lattices associated with the presheaf

$$\langle \{\mathbf{S}(\mathbf{D}/\alpha(a))\}_{a \in C(X)}, \{\mathbf{S}(\rho_{ab})\}_{a \subseteq b} \rangle.$$

The continuous global sections of $\langle \mathcal{S}, \pi, X \rangle$, with the operations defined pointwise, form a Stonean residuated lattice $\mathbf{A} = \mathbf{A}(\langle \mathbf{B}, \mathbf{D}, \phi \rangle)$.

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$$\mathbf{T}(\mathbf{A}) \cong (\mathbf{B}, \mathbf{D}, \phi).$$

Conclusion

The category of distributive Stonean residuated lattices is equivalent to the category $\mathcal{T}$ of triples.

Work in progress

(joint work with M. Marcos and S. Ugolini) Generalize the results for a category of distributive residuated lattices with an MV-retraction.

- Aguzzoli, S., Flaminio, T. and Ugolini, S, *Equivalences between the subcategories of MTL-algebras via boolean algebras and prelinear semihoops*, Journal of Logic and Computation, 2017, https://doi.org/10.1093/logcom/exx014.
- Chen, C. C. and Grätzer, G., Stone Lattices. I: Construction Theorems, Canad. J. Math. *21* (1969), 884–994.
- Katriňák, T., *A new proof of the construction theorem for Stone algebras, Proc. Amer. Math. Soc.*, **40** (1973), 75–78.
- Maddana Swamy, U. and Rama Rao, V. V., *Triple and sheaf representations of Stone lattices*, Algebra Universalis **5** (1975), 104–113
- Montagna, F. and Ugolini, S., *A categorical equivalence for product algebras*, Studia Logica **103** (2015), 345-373.

Thanks for your attention