What kind of bonus point system makes the rugby teams more offensive?

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Motivation

- Rugby Union is a game in constant evolution
- Experimental law variations every year
- Different score systems in the world

History

Brocas - Carrillo use Game Theory to find some results on the strategies of football teams depending on the score system

Goals

- Use a static model to compare the level of "offensiveness"
- Use a dynamic model to compare the possible payoffs

Model

- 2 teams $i \in \{A, B\}$
- Events $(a, b) \in N \cup 0xN \cup 0$
- Equivalence relations between events
- Strategies $(\theta^A, \theta^B) \in [\underline{\theta}, \overline{\theta}]^2$
- Team A wins if it scores more tries than team B (no penalty kicks or conversions)

Model (cont.)

- $\alpha(\theta^A, \theta^B)$: probability of team A scoring a try
- $\beta(\theta^A, \theta^B)$ probability of team A receiving a try
- Three different score systems:
  - (NB) 4 points for the victory, 2 for a tie and 0 for losing
  - (+4) 1 extra point for scoring 4 or more tries, and 1 extra point if the losing team loses by only 1 try of difference
  - (3+) 1 extra point if the winning team scores 3 tries more than the losing team, and 1 extra point if the losing team loses by only 1 try of difference

Properties of the probability functions

- $\alpha_1 > 0, \alpha_2 > 0, \alpha_{11} \leq 0, \alpha_{22} > 0$
- $\beta_1 > 0, \beta_2 > 0, \beta_{11} > 0, \beta_{22} \leq 0$
- $\alpha_{12} = \beta_{12} = 0$

Utility function

- $U^i((\theta^A, \theta^B), (a, b)) = \alpha(\theta^A, \theta^B) f_i(a+1, b) + (1 - \alpha(\theta^A, \theta^B) - \beta(\theta^A, \theta^B)) f_i(a, b) + \beta(\theta^A, \theta^B) f_i(a, b+1)$
- $f_i(a, b)$ depends on the score system used
- Find the Nash Equilibrium in every score system and compare the equilibria

Example 1

Event (0,0)

(+4) and (3+) System

$U^A = \alpha 4 + (1-\alpha-\beta)2 + \beta$

$U^B = \alpha + (1-\alpha-\beta)2 + \beta 4$

The Nash Equilibrium is given by $\dfrac{\beta_1}{\alpha_1} = 2$ and $\dfrac{\alpha_2}{\beta_2} = 2$

(NB) System

$U^A = \alpha 4 + (1-\alpha-\beta)2$

$U^B = (1-\alpha-\beta)2 + \beta 4$

The Nash Equilibrium is given by $\dfrac{\beta_1}{\alpha_1} = 1$ and $\dfrac{\alpha_2}{\beta_1} = 1$

Example 2

Event (3,1)

(+4) and (3+) System

$U^A = \alpha 5 + (1 - \alpha - \beta) 4 + \beta 4$

$U^B = \beta 1$

The Nash Equilibrium is given by $(\overline{\theta}, \overline{\theta})$

(NB) System

$U^A = 4$

$U^B = 0$

The Nash Equilibrium is given by $(\widetilde{\theta}^A, \widetilde{\theta}^B)$

Example 3

Event (5,3)

(+4) System

$U^A = 5$

$U^B = \beta 2$

The Nash Equilibrium is given by $(\widetilde{\theta}^A, \overline{\theta})$

(3+) System

$U^A = \alpha 5 + (1 - \alpha - \beta)4 + \beta 4$

$U^B = \beta 1$

The Nash Equilibrium is given by $(\overline{\theta}, \overline{\theta})$

(NB) System

$U^A = 4$

$U^B = 0$

The Nash Equilibrium is given by $(\widetilde{\theta}^A, \widetilde{\theta}^B)$

Results

- Pairwise comparisons indicate that the $(+4)$ system is better than the other score systems
- There is no difference between the $(3+)$ and (NB) systems

Dynamic Model

- Equilibrium Payoffs of Dynamic Games (Massó - Neme (1995))
- Stochastic Games with transition function 0 or 1
- The characterization is in terms of stationary strategies

Dynamic Model

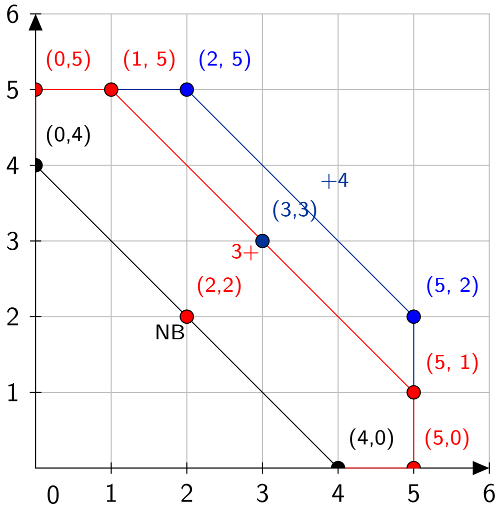
- $G = ((W, w^1), [\underline{\theta}, \overline{\theta}]^2, T)$
- $W = \{(a, b)\}_{(a,b) \in N \cup 0 x N \cup 0}, \; w^1 = (0, 0)$
- T is a deterministic transition function
- All our strategies are stationary (do not depend on the history)

Feasible Payoffs (Massó- Neme (1995))

Theorem

A payoff $v \in \mathbb{R}^n$ is feasible if and only if there exists a set of connected stationary strategies such that v is a convex combination of the utility of each of those strategies

Feasible Payoffs



Conclusions

- The (+4) systems is better in the two settings
- Rather surprisingly, there is no difference in the static model between (3+) and (NB)
- The difference appears in the dynamic model

Future Work

- Find the equilibrium payoffs and compare the three score systems